

IS THE CONVENTIONAL VALUE-AT-RISK (VAR) MODEL AN APPROPRIATE TOOL FOR ESTIMATING MARKET RISK? THE CASE OF AN INDICATIVE JAMAICAN FINANCIAL INSTITUTION¹

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ABSTRACT

The main objective of this paper is to highlight pitfalls in using a conventional VaR model to estimate market risk for a Jamaican financial institution. The conventional VaR model for an indicative Jamaican bank (Bank A) was assessed for accuracy and validity using a backtesting procedure.³ With respect to validity, the conventional model's underlying assumption of a normal distribution of loss returns over the period 3 January 2007 to 12 September 2008 was found to be inappropriate due to the non-normal distributional tendencies in most of the bank's asset classes. In terms of accuracy, the occurrence of a relatively large number of backtesting exceptions implied that there was a breakdown in bank A's VaR model over the review period and hence the need to explore alternative measures. In this regard, the paper recommends the use of Archimedean copulas, given their flexibility and ability to capture tail events as well as non-linear dependence structures. These properties render Archimedean copula risk models more appropriate for estimating (tail) risk than conventional VaR models, especially in relation to capital allocation decisions. Of the three Archimedean copulas contemplated, the Gumbel-Hougaard model provided the most conservative quantile risk estimates, as evidenced in the least number of backtesting exceptions.

JEL Classification: C15, C52, C63, C65, G32

Keywords: Value-at-Risk, Expected Shortfall, backtesting, Archimedean copulas, tail events, loss distribution

¹ Conventional in this context refers to the assumption of a normal distribution of returns.

² The views expressed are the author's and not necessarily those of RBTT Bank Jamaica Limited.

³ The paper uses actual data for an indicative regulated financial institution, which will remain anonymous for confidentiality and regulatory reasons.

1.0 Introduction

1.1 *The Jamaican Banking Sector*

Ever since the fall-out in the banking sector arising from the crisis in the mid-1990s, financial institutions in Jamaica have considerably enhanced their risk management practices. There has also been significant consolidation in the sector a consequence of the establishment of the Financial Sector Adjustment Company (FINSAC) in 1997 to resolve the solvency and liquidity problems associated with the high levels of non-performing loans on the balance sheets of troubled banks. The high levels of non-performing loans were largely due to high inflation and insufficient regulatory controls, two of the major contributors to the banking crisis (Bank of Jamaica 2004).

Currently, there are seven commercial banks in operation, all of which subscribe to international regulatory standards for supervised banks in one form or another. As part of their regulatory oversight, the Bank of Jamaica (BoJ) encourages banks to subscribe to internal risk management guidelines as per the Basel core Principles for Regulated Banks (i.e. Basel I and Basel II). This includes *inter alia* the monitoring of financial soundness indicators as well as the development of internal risk models for the management of market, credit and operational risks. Implementation of Basel's guidelines is not only a compliance issue, it is a significant opportunity for banks to achieve a competitive advantage and an additional source of funds as markets are more willing to invest in businesses offering highest return per measure of risk (Lebedev 2008).

1.2 *Rationale and Structure of Paper*

The global economic crisis has had a significant impact on the environment in which Jamaican banks operate, to the extent that most institutions have intensified marketing strategies as well as conducted corporate restructuring and product rationalization exercises. The increasing levels of delinquency and liquidity constraints affecting the banking sector recently can be attributed to the insufficiency of existing risk management tools to accurately estimate market risk exposures. This was glaringly manifested in the large number of banks that experienced, and were unable to sufficiently meet margin calls during the December 2008 quarter. Further, a sharp decline in prices on Government of Jamaica (GoJ) denominated assets during 2008 resulted in significant mark-to-market losses in the investment portfolios of some of the major local financial institutions. These events were hinted at some years earlier in an Standard and Poor (S&P) article, which stated the following;

.....*VaR has some severe limitations that, if not properly appreciated, can lull a company into a false sense of security. For instance, VaR lacks the criteria to provide a consistent measure for comparing the relative risk appetite across institutions, as the assumptions used by firms in calculating VaR can be vastly different and have varying degrees of precision. In addition, as a stand-alone measure, VaR ignores the extent of tail risk that an institution is exposed to, especially under abnormal market conditions, and falls short of satisfying a key mathematical property required of a robust measure of risk. For these reasons, Standard & Poor's and other market analysts believe that VaR should be interpreted with caution in evaluating market risk and should ideally be used in conjunction with other risk measures.....*⁴

In light of the above excerpt, the main contribution of this paper is to show how the validity of market-risk estimates for an indicative Jamaican bank can be improved if the conventional VaR model is replaced with an Archimedean copula-based VaR model. Bank A was chosen for the analysis given its long-standing prominence and reputation as an innovative Jamaican financial institution one that is diversified across business lines as well as across regions/countries. Also, the fact that bank A offers a wide variety of products and services (both banking and non-banking) makes it an appropriate choice for this analysis as an indicative bank one whose balance sheet to a large extent reflects the dynamics of the Jamaican banking system.

The rest of the paper is structured as follows; section two explores the characteristics, guidelines, industry standards and caveats of VaR as an estimate of market risk. Section three seeks to invalidate bank A's conventional VaR model using a backtesting framework. Sections four and five present a more appropriate model framework for the estimation of the bank's market risk, specifically through the use of copula-derived VaR and *Expected Shortfall* (ES) estimates. Section six gives conclusions and recommendations arising from the analyses.

2.0 The Measurement and Management of Market Risk

2.1 Characteristics of Value-at-Risk (VaR)

Market risk may be defined as the adverse dollar-impact on the mark-to-market value of an investment portfolio in response to financial market disturbances. Some of these disturbances include

⁴ Excerpt from Standard and Poor's article: "Chasing Their Tails: Banks Look beyond Value-At-Risk" published 12 July 2005.

volatility in interest rates, changes in the slope or shape of yield curves, widening of credit spreads, volatility in equity indices, shocks to exchange rates and volatility in commodity and energy prices.

VaR is one of the most widely accepted appraisals of market risk. It is an estimate of the worst loss (negative return) that a security or portfolio of securities can incur over a fixed time period in the future, at a specified probability.⁵ The principle of VaR assumes that the portfolio being considered is fixed over the time horizon in question. In other words, no risk-mitigating action can be taken to reduce the exposure of the portfolio in the event of adverse movements in market risk factors. This assumption becomes more unrealistic as the time horizon increases. In addition, the reliability of VaR estimates is dependent on their associated horizons, length of the returns time series used in the estimation and idiosyncrasies of the assets or asset classes contained in a portfolio. At the very least, this raises questions about a “one size fits all” approach to VaR estimation.

2.2 *International Guidelines on VaR estimation*

There are three approaches used to estimate VaR; Historical - uses historical or empirical data, Parametric - assumes a normal distribution of returns and Monte Carlo - involves Monte Carlo simulations of a loss distribution (Jorion 2001). The Basel Committee for banking supervision recognizes all three approaches, subject to a few guidelines for regulated banks. For the purposes of this discussion, three relevant guidelines for VaR estimates are as follows:

1. VaR estimates must be based on a sample size of at least one year of daily returns (roughly 250 trading days) or a weighted sample with an average lag of no less than six months,
2. VaR estimates must be based on a 0.01 one-tailed probability (99th percentile, one-tailed confidence interval) and,
3. Banks' market risk exposures are to be estimated for a 10-day holding period. Basel allows 1-day VaR estimates to be scaled up using the “square-root-of-time rule”. This follows from the assumption that portfolio returns are serially independent and have a normal distribution.

2.3 *Estimating Market Risk using VaR*

2.3.1 *Industry standard*

Although VaR is an industry-standard measure of market risk, it is a generic term. Institutions may use different returns distributions, confidence levels and/or horizons in their risk management frameworks. The historical VaR (empirical) approach uses historical returns to estimate VaR. In this

⁵ For example, if today you are told that the 99%, 1-day VaR of a portfolio is \$50,000, it means that there is a 1% chance that at the end of trading tomorrow (assuming tomorrow is not a holiday, Saturday or Sunday), this portfolio could incur a loss in excess of \$50,000.

estimation process however, previous market upheavals may be excluded (when relatively shorter time series are used) or the effect of more recent market events may be diluted (when relatively longer time series are used). Although conventional sampling theories⁶ suggest that large samples are better than small ones, its applicability in estimating historical VaR can be dangerously deceptive.⁷ Risk managers are generally more concerned with the likely state of affairs tomorrow or next week rather than next year (Hoppe 1998). Therefore to give greater prominence to more recent events whilst preserving the econometric validity of the data, risk managers may opt to apply exponential weights (decay factors) to long time series.

Financial institutions may alternatively choose to model risk exposures on the basis of approximating future changes in risk factors as a normal distribution via historical volatilities and correlations (i.e. the variance-covariance or parametric VaR approach) or by simulating more complex distributions for the risk factors from loss distributions (Monte Carlo VaR approach). The Monte Carlo VaR approach is widely viewed by risk practitioners as the most practical measure of VaR, especially in a context where market volatility has become more pronounced in recent times. If the distribution of returns (losses) is assumed to be known, then very precise probability statements can be made about quantiles of its loss distributions.

Estimating the VaR of a portfolio of securities (portfolio VaR) generally involves computing a percentile of an aggregated distribution (or joint distribution) of portfolio-constituent returns. The rationale of this approach is to capture the correlation and diversification effects associated with a portfolio of financial assets. A *copula* is one such mechanism used to express the joint distribution of two or more variables (returns). Each return may be modeled using parametric marginal distributions (such as a normal distribution) or using stochastic processes (such as ARMA/GARCH models) in order to explain the level and (conditional) volatility of returns. Importantly, copulas allow for each return in a portfolio to be modeled as non-identical, fat-tailed parametric distributions (e.g., Asset 1 could be modeled as Pareto, Asset 2 as Lognormal, Asset 3 as Weibull, etc). Copulas also allow risk practitioners to account for linear and non-linear dependencies amongst portfolio-constituent returns, particularly in the tails of the loss distributions.

2.3.2 Caveats

In the Jamaican context, the ability to make VaR statements with high confidence depends on the robustness of a bank's internal VaR model to the extent that returns distributional assumptions are

⁶ For e.g., the *Law of Large Numbers* and the *Central Limit Theorem*.

⁷ This is abstracting from issues associated with the relations between statistical power, effect size and Type I errors.

violated, specifically in the tails of the distributions. Since VaR estimates are typically concerned with extreme probabilities (e.g. 1%, 0.1%) and given the increased occurrence of “tail events” in financial markets, the assumption of a normal distribution of returns is unrealistic. Various studies have shown that financial returns are characterized by “fat-tails” (high kurtosis), asymmetry (non-zero skew coefficient), volatility clustering, serial correlation and non-parametric multivariate distributions (Hotta *et al.* 2006). This is also the case for Jamaican financial markets as is shown in Table 1 in the section 3.3. These characteristics make it difficult to fit financial returns data with “well-behaved” parametric distributions (Hotta *et al.* 2006). Consequently, it is not only difficult but also inappropriate to fit returns data to the widely acceptable normal distribution. This does not bode well for the validity of bank A’s VaR model, given its underlying assumption of a normal distribution of returns as per the third Basel guideline in section 2.2 above.

Estimating portfolio VaR under the assumption of identical, normally distributed returns (i.e. using a *Gaussian* copula) might therefore not only be challenging but invariably inaccurate. Empirical evidence has proven that the normal-distribution-of-returns assumption is inappropriate because it underestimates both the thickness of the tails, as well as the dependence structure of a portfolio of returns (Romano 2004). Consequently, VaR derived from Gaussian copulas tend to understate possible portfolio losses.⁸ This is where dynamically customized stress tests must become a critical part of a bank’s risk management framework, given ongoing innovations in financial products and market dynamics.

3.0 Validating Bank A’s VaR Model: A Backtesting Approach

3.1 Validation of VaR Models: Backtesting

The use of VaR in a real-world context without a clear understanding of the implications of returns distribution assumptions can lead to erroneous conclusions about the exposure of an institution to market risk. In light of this as well as the caveats associated with the estimation of portfolio VaR, it is important that banks establish a framework for periodic validation of risk models. It should be noted that VaR estimates produced by any risk model is unable to accurately quantify the worst-case loss that could occur as a result of extreme, unusual or unprecedented market conditions. These extreme circumstances are captured only in user-defined stress tests or through the use risk measurement software with built-in mechanisms such as economic scenario generators (ESGs).

⁸ The calculation of portfolio VaR using Gaussian (normal) copulas is supported by many risk management applications due to the convenient assumption of a multivariate normal distribution of returns.

The concept of backtesting is widely used by risk practitioners to validate the accuracy and appropriateness of risk models. Backtesting a model that produces a 10-day VaR estimate daily for example, would involve a comparison of the 10-day VaR at the close of each trading day (for a minimum of 100 days) with the actual change in portfolio value over successive 10-day periods.⁹ A backtesting exception is said to occur if on a given day, the actual 10-day mark-to-market change in portfolio value is negative and exceeds the 10-day VaR estimate. So for a 99%, 10-day VaR, a backtesting exception should be expected once every 100 days on average.

3.2 Methodology

Daily financial returns based on the composition of bank A's portfolio over the period January 3, 2007 to September 12, 2008 (339 observations) as well as 10-day Monte Carlo VaR estimates from the bank's internal VaR model are used to conduct the backtesting procedure. The review period primarily captures a phase of relative stability in the Jamaican financial markets (January 2007 to June 2008) and to a lesser extent a phase of significant volatility and adverse movements in market risk factors (July 2008 to September 2008). The latter phase coincides with the embryonic stages of the global financial crisis. The overall period chosen for the analysis therefore minimizes the influence of "events" on the number of backtesting exceptions.

Bank A's model assumes an exponential scaling factor of 0.94 and that returns are normally distributed. It utilizes a Gaussian copula to compute 1-day Monte Carlo portfolio VaR estimates. Nine asset classes were considered and the daily mark-to-market gains/losses for each asset class were used to calculate the bank's 10-day portfolio mark-to-market gains/losses. By virtue of the square-root-of-time rule, the 1-day VaR estimates were scaled-up by a factor of $\sqrt{10}$ to get the 10-day VaR estimates. Bank A's 10-day VaR estimates were then compared to actual changes in its portfolio market value over successive 10-day horizons for each trading day during the review period.

3.3 Results

Bank A's internal VaR model was not able to accurately predict mark-to-market portfolio losses for the review period. The backtesting procedure revealed that the percentage of backtesting exceptions for the portfolio was 29.2%, significantly exceeding the 1.0% prescribed limit according to the 99th percentile subsumed in the VaR estimates. This is not surprising given the sharp movements in market risk factors

⁹ These profits (or losses) normally exclude non-trading revenues such as commissions & fees and revenues from intraday trading.

over the review period, which translated into an even further deviation from the assumption of a normal distribution of returns.

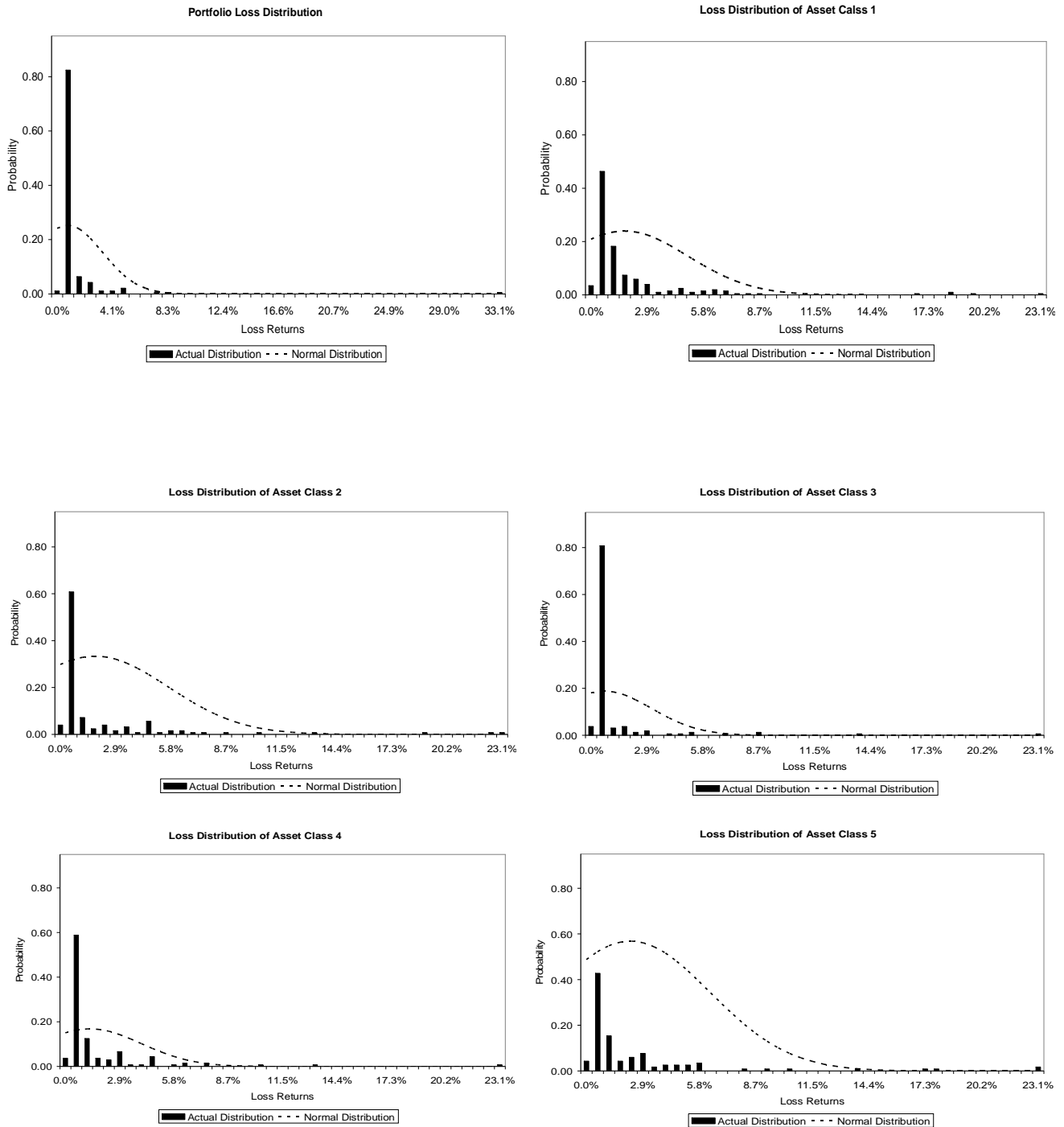
Table 1: Select Statistics for Bank A's Loss Distribution Returns: 3 January 2007 to 12 September 2008

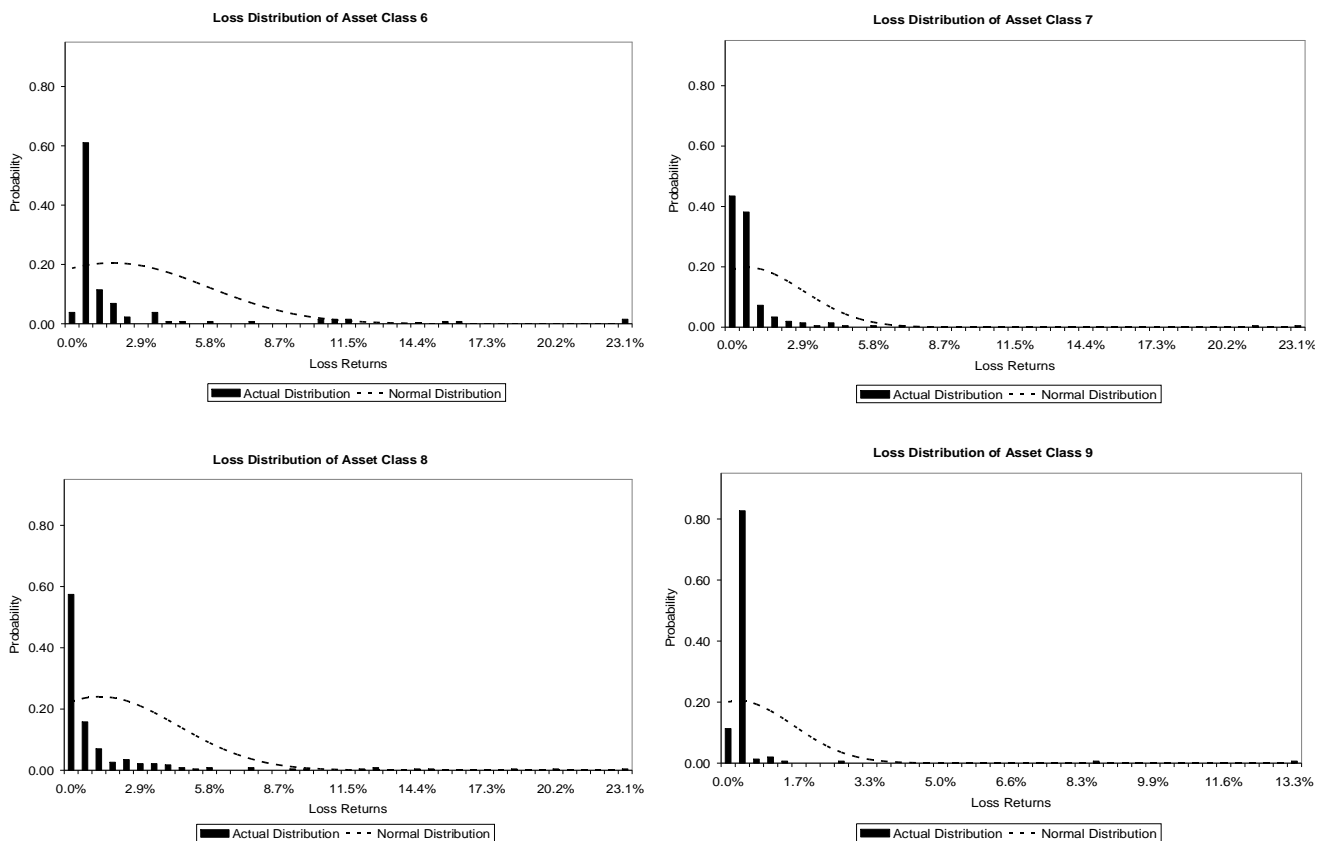
	Overall Portfolio	Asset Class 1	Asset Class 2	Asset Class 3	Asset Class 4	Asset Class 5	Asset Class 6	Asset Class 7	Asset Class 8	Asset Class 9
Mean	0.008	0.017	0.010	0.075	0.007	0.004	0.006	0.014	0.026	0.015
Standard dev	0.026	0.033	0.022	0.268	0.015	0.008	0.014	0.058	0.072	0.087
skewness	10.491	4.113	3.782	6.413	4.916	3.470	3.584	7.980	4.175	8.706
kurtosis	127.080	19.710	16.392	49.610	32.120	13.312	13.775	71.221	19.402	79.923
Jarque-Bera	124,049.0	2,934.0	1,232.1	15,677.6	5,352.9	753.3	914.2	42,339.3	3,218.2	38,877.1
Chi squared Statistic	4	2,819.670	3,499.204	1,442.593	17.067	6.621	6.814	1.521	11,284.243	1,138,079.456
n	188	203	125	161	136	117	131	207	228	150
Critical Chi (99%, k-1 d.o.f.)	22.164	22.164	22.164	22.164	22.164	22.164	22.164	22.164	22.164	22.164
Critical Chi (95%, k-1 d.o.f.)	26.509	26.509	26.509	26.509	26.509	26.509	26.509	26.509	26.509	26.509
Is Return Normally Dist at the 99% Level?	Yes	No	No	No	Yes	Yes	Yes	Yes	No	No
Is Return Normally Dist. At the 95% Level?	Yes	No	No	No	Yes	Yes	Yes	Yes	No	No
max	0.331	0.231	0.133	2.530	0.125	0.044	0.084	0.578	0.513	0.887

Source: Author's Calculations

In light of the breakdown in the bank's VaR model, an investigation into the distributional properties of the loss returns data was conducted. The distributions of loss returns for all asset classes were assessed using the Jarque-Bera and Chi-Squared normality tests. The results in Table 1 above show that the Jarque-Bera test unequivocally rejected the null of normality for all series. The Chi-Squared test was less convincing as it rejected the null of normality only for Asset Classes 1,2,3,8 and 9 at the 95% and 99% confidence levels (see Table 1). However, tail events were evident in most of the series implying fat-tailed loss distributions (see Figure 1 below). Also, all asset classes except Asset Class 5 evidenced a higher than "normal" kurtosis value. These characteristics are not only supported by the Jarque-Bera test's result of a deviation from normality in all asset classes, they also point to lognormal distributional tendencies.

Figure 1: Actual Loss Distributions for Bank A's Asset Classes vis-à-vis the Normal Distribution





Source: Author's calculations

4.0 Alternatives to the Conventional VaR Estimate

As highlighted in the backtesting procedure above, there was a significant number of backtesting exceptions, as the model was unable to quantify the "real-world" exposure of bank A to market risk. For all asset classes, the percentage of backtesting exceptions far exceeded the 1% threshold implied by the 99th percentile used in the bank's VaR calculations. In other words, the VaR underestimated the extent to which Bank A's portfolio was exposed to mark-to-market losses. The large number of backtesting exceptions could have been attributable to the conventional model's inability to capture extreme events given the non-normal nature of most asset classes contemplated. It therefore stands to reason that more conservative and coherent measures of risk such as *Expected Shortfall (or Tail-VaR)* and estimates derived from *Archimedean* copulas could produce better estimates.

4.1 Expected Shortfall (ES)

The ES for a security or portfolio of securities is the average loss that may be incurred in the event that the VaR level is exceeded. Advocates of ES are justified by the fact that unlike VaR it satisfies the sub-additive

axiom for coherence and captures tail risk.¹⁰ The calculation of ES incorporates the shape of the conditional distribution of the worst-case scenarios beyond a specified quantile. Therefore it is a more conservative and robust measure of market risk and to that extent more appropriate for capital allocation decisions. Similar to VaR though, ES is based on a particular quantile and (future) time horizon.

4.2 Archimedean Copulas

With respect to estimating portfolio risk, Archimedean copulas have become increasingly popular in actuarial science and financial risk management (Wu et al. 2006). These types of copulas are characterized by a single-valued generator function, thereby eliminating the need for high-dimensional, cumbersome multivariate functions, as is the case for other families of copulas. Importantly, unlike their Gaussian counterpart, Archimedean copulas do not understate the thickness of the tails of returns data since it accommodates the use of “fat-tailed” marginal distributions. Archimedean copulas are also relatively easy to implement and are considered suitable for capturing the dependence structures of a wide range of financial returns (Wu et al. 2006).

5.0 Application of Archimedean Copulas: VaR and Expected Shortfall

The same data for Bank A were applied to three multivariate Archimedean copulas; Gumbel-Hougaard, Clayton and Frank.¹¹ This was done with the view of determining whether copula-based risk estimates would have resulted in a lower number of backtesting exceptions. To preserve consistency with the bank’s conventional VaR model, 99%,1-day VaR and ES estimates were computed for each of the copula-based risk models using the simulation algorithm in the Appendix. These measures were then scaled-up by a factor of $\sqrt{10}$ as per the square-root-of-time rule. A comparison with the 10-day mark-to-market losses for bank A was then conducted, similar to that which was done for bank A’s conventional VaR model.

Inputs for the copula-based risk models include a dependence parameter derived from Kendall’s Tau correlation coefficient for each pair of returns series (Cliff and Charlin. 1991).¹² For flexibility, the model allows the risk manager/analyst to choose different types of marginal distributions to fit each return series.¹³ The model also allows for variation in the decay factor, as well as the number of simulations in the

¹⁰ The sub-additive axiom says that the risk of a diversified portfolio is always less than the risk of the portfolio’s individual components.

¹¹ The Appendix shows an algorithm that may be used to simulate multivariate Archimedean copulas. VaR and expected shortfall can be estimated using Monte Carlo simulations of the returns generated using the algorithm.

¹² Kendall Tau’s correlation assesses the degree of dependence ordinally or where there is a monotonic but not necessarily linear trend in data. It is robust to the nature of the distributions being considered and is relatively insensitive to extreme values.

¹³ Four types of distributions are available in the model; Lognormal, Weibull, Normal and Pareto.

estimation process. For this aspect of the research, the three copula models assumed 10,000 simulations, a decay factor of 0.94 and a lognormal distribution function for each of the returns series.

5.1 Backtesting Results for Alternative Portfolio Risk Estimates

Gumbel-Hougaard Copula

Backtesting results for the Gumbel-Hougaard copula risk model revealed that relative to the 29.2% for the conventional VaR model, backtesting exceptions decreased to approximately 5% and 3% for the VaR and ES estimates, respectively. This was the lowest amongst the three copula models considered and is not surprising, considering that VaR and ES estimates tend to be highest for the Gumbel-Hougaard copula (Wu et al. 2006). Therefore, this copula provided the most conservative measure of risk for bank A.

Clayton Copula

Backtesting results for the Clayton copula risk model revealed that relative to the 29.2% for the conventional VaR model, backtesting exceptions decreased to approximately 9% and 6% for the VaR and ES estimates, respectively. Again this was not surprising since VaR and ES estimates tend to be lowest for the Clayton copula relative to the other copulas (Wu et al. 2006). Therefore, this copula provided the least conservative measure of risk for bank A.

Frank Copula

Backtesting results for the Frank copula risk model revealed that relative to the 29.2% for the conventional VaR model, backtesting exceptions decreased to approximately 7% and 5% for the VaR and ES portfolio estimates, respectively.

6.0 Conclusion and Recommendations

The main objective of this paper was to highlight the pitfalls in using VaR as a measure of market risk for an indicative Jamaican financial institution, Bank A. Results of the backtesting procedure revealed that there was a breakdown in bank A's internal VaR model over the review period, as evidenced in the relatively large number of backtesting exceptions. This was in a context where there has been heightened volatility in financial markets in recent times, thereby invalidating the conventional VaR model's assumption of normality.

The assumption of a multivariate normal distribution is standard in many risk measurement applications. The fact that these distributions are defined by a few parameters makes them easy to

implement in a Monte Carlo simulation framework. The empirical evidence in this paper has proven that the multivariate normal distribution assumption is inappropriate because it underestimates both the thickness of the tails, as well as the dependence structure of bank A's portfolio constituents. Therefore standard risk measurement applications may have a tendency to understate portfolio market risk for some Jamaican financial institutions given their increased appetite for sophisticated investment products with non-linear correlation and pricing structures. The understatement of market risk in these products via conventional VaR models has adverse implications for financial institutions' capital allocation decisions and by extension their obligation to maximization shareholder value.

Results of the comparison between the portfolio risk estimates of bank A's conventional model and the copula risk models showed that the number of backtesting exceptions was significantly lower for the three pairs of copula risk estimates. This was largely attributable to the use of fat-tailed parametric marginal distributions and a simulation algorithm that utilized a non-linear dependence parameter. The Gumbel-Hougaard model seemed to provide the most conservative measures of risk, as it produced the least number of backtesting exceptions over the review period.

Based on the findings of the backtesting procedure, copula-derived VaR and ES estimates should be considered as more appropriate measures for market risk for Jamaican financial institutions. This paper recommends the use of Archimedean copulas, which have become increasingly popular in actuarial science and financial risk management. Proponents of Archimedean copulas are justified by the fact that they are relatively easy to implement and possess many flexible and user-friendly properties that make them suitable for modeling various dependence structures and asset classes.

In terms of further work, the paper could be extended to determine whether the number of backtesting exceptions could be even further reduced by explicitly computing 10-day VaR and ES estimates using 10-day returns as opposed to daily returns and the square-root-of-time rule. Also, other types of non-parametric distributions and/or stochastic processes could be contemplated to fit each return more accurately, thus improving the model's ability to estimate market risk.

APPENDIX

Alternative Measures of Portfolio (Market) Risk: Archimedean Copulas

Let φ be a function such that $\varphi : [0, 1] \rightarrow [0, 1]$. We will call this function an Archimedean generator if it satisfies the following three conditions (see Chapter 4 of Nelsen (1999));

1. $\varphi(1) = 0$,
2. φ is monotonically decreasing and
3. φ is convex.

Let $u = (u_1, \dots, u_n)'$ be an n -dimensional unit vector with $u_k \in [0, 1]$ for all $k = 1, \dots, n$. A copula C is termed Archimedean if there exists a generator function φ such that C has the form;

$$C(u) = \varphi^{-1}(\varphi(u_1) + \dots + \varphi(u_n));$$

where φ^{-1} denotes the inverse function of the generator.

Notice that if the first derivative, φ' , exists, then by condition (2), we must have $\varphi' \leq 0$, and if the second derivative, φ'' , exists, then by condition (3), we must have $\varphi'' \leq 0$. For our purposes, we consider only Archimedean generators which are continuous and whose higher derivatives exist. For C to be an n -dimensional copula, φ^{-1} must be completely monotonic such that if all derivatives exist, we must have...

$$\frac{(-1)^k d^k \varphi^{-1}(u)}{du^k} \geq 0, \text{ for } k = 1, 2, \dots, n.$$

In an (Archimedean) copula context, the distribution function of any continuous random variable has a Uniform $U(0, 1)$ distribution such that;

$$F_{s_k}(s) = s_k, \text{ for } s \in (0, 1), k = 1, 2, \dots, n-1 \text{ and}$$

$$F_T(t) = \frac{1}{(n-1)!} \times \int_0^t \varphi^{-1(n)}(w) [\varphi(w)]^{n-1} \varphi'(w) dw$$

Thus, in order to generate an n -tuple vector (X_1, \dots, X_n) with an Archimedean copula, the procedure is as follows;

1. Generate n independent $U(0, 1)$ random variables. Denote them by w_1, \dots, w_n .
2. For $k = 1, 2, \dots, n-1$, set $s_k = w_k^{1/k}$
3. Set $t = F_T^{-1}(w_n)$ and solve for t .

4. Set $u_1 = \varphi^{-1}(s_1 \dots s_{n-1} \cdot \varphi(t))$,

$$u_k = \varphi^{-1}\left((1 - s_{k-1}) \prod_{j=k}^{n-1} s_j \cdot \varphi(t)\right), \text{ for } k = 2, \dots, n, \text{ and}$$

$$u_n = \varphi^{-1}((1 - s_{n-1}) \cdot \varphi(t))$$

The simulated returns are $x_k = F_k^{-1}(u_k)$ for $k = 1, 2, \dots, n$, where $F_k^{-1}(\cdot)$ is the inverse of the cumulative distribution function for asset class k 's returns.

The Archimedean Copula generator functions are as follows;

a. Clayton Copula

$$\varphi(u) = \frac{u^{-\theta} - 1}{\theta}, \theta > 0$$

b. Frank Copula

$$\varphi(u) = -\log\left(\frac{e^{-\theta u} - 1}{e^{-\theta} - 1}\right), \theta \neq 1$$

c. Gumbel-Hougaard Copula

$$\varphi(u) = (-\log u)^{1/\theta}, \theta > 1$$

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