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Stylised Facts of the Business Cycle in Barbados

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Une analyse des faits stylisés du cycle barbadien
Stylised facts of the business cycle in Barbados

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Abstract: This paper examines the characteristics of the business cycle in Barbados for the quarterly period 1974-2003 using several adaptive and non-adaptive filters. The business cycle is described and explained and a prediction of the dynamic linkages among principal macroeconomics variables is made.

Résumé: L'objet de cet article est l'examen de la problématique du cycle économique dans une petite économie ouverte en développement, la Barbade. S'inscrivant dans une dimension essentiellement empirique, il fait d'abord le point sur les méthodes qui composent aujourd'hui la "boîte à outils" de l'analyse cyclique moderne. Il discute ensuite des résultats d'application de ces techniques économétriques aux principales variables de l'économie barbadienne.
Les principaux enseignements sont de plusieurs ordres. D'abord…

I. Introduction

Macroeconomic time series are affected by different causal factors that could influence the incidence of turning points in the trajectories of these variables. Indeed, to estimate these turning points, business cycle analysis, which involves the description, explanation and prediction of the dynamic linkages among the principal macroeconomic variables in the short term, is the approach most often used in the empirical literature.

The recent literature conceptualises the business cycle as a phenomenon comprising alternating periods of growth and decline in economic activity. These irregular, but recurring, changes in the level of economic activity, which are mainly guaged by looking at fluctuations in GDP, are driven by changes in other variables. Put simply, the measurement and identification of cyclical movements implies the separation of the trend and the cyclical components. The resulting detrended series becomes a critical component in the formulation of economic policy, as it is useful in choosing between structural and cyclical public policy. Given that economic disequilibria, such as unemployment and budget deficits, may be of either a transitory or a persistent nature, the ability to identify the optimal point in an economy's cycle is therefore an advantage in the formulation of plans for remedial action to minimise such imbalances. The decomposition of business cycles may be approached using both univariate and multivariate methods. In the first case, the aim is to provide a general description of indicators such as periodicity, amplitude, volatility, etc. In the latter approach, the cyclical component of the GDP series is compared with those of the principal explanatory variables, in order to garner information on their differences and similarities.

In light of all the issues outlined above, a central question emerges: how can one decompose raw data into trend, cyclical and irregular components? Following Nelson and Plosser (1982), the techniques used to estimate the trend and the cycle can lead to erroneous results if they are not adapted to take into account the statistical properties of the series under consideration. For instance, applying the method of linear regression over time in order to estimate the trend generally gives a non-stationary trend, which is practically useless, since the GDP series is most often integrated of the first order.

One can therefore understand why the quest for reliable solutions to this very delicate problem has so completely absorbed the interest of so many economists. Though the list is by no means precise or exhaustive, it is fair to say that there are currently dozens of alternative approaches to trend/cycle decomposition, which may be classed according to two broad categories. The first encompasses adaptive filtering techniques, which 'learn' the characteristics of the series as it moves along its path. The other consists of non-adaptive filtration, which, conversely, is based on pre-calculated fixed-coefficient filters.

This paper forms part of a larger body of work dedicated to the study of business cycles in small developing countries, with specific reference to Caribbean countries. While business cycle trends are in effect well documented for the developed countries, both in terms of their theoretical modelling and their empirics, it must be noted that such work are rare in the case of developing countries, as indicated by Agénor, McDermott and Prasad (2000). As such, the objective of this paper is to assist in filling this void by analysing the stylised facts of the business cycle in Barbados, a typical Caribbean economy.

After the introduction, a description and definition of the Barbadian business cycle is given. Next, an overview of the methodological principles are presented, followed by a discussion of the data. Results concerning the trends and cycles of the sectoral production are next in line, then the trend and cyclical interaction of GDP with other macroeconomic variables are assessed. The final part is the conclusion.

trend, in the process diverting considerable financial resources from the rest of the economy, while tourism suffers a decline in arrivals from regional sources. Consequently, the Barbadian economy records declines in real GDP of 3.1% in 1990, then 4.1% in 1991 and 6.2% in 1992. This real sector crisis is accompanied by a balance of payments crisis, which leads to capital flight and debt accumulation.

- After 1993, and with the application of austerity measures, Barbados resumed a positive growth path, with real GDP rising above 3% between 1996 and 1997.
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Figure 1. Annual Percentage Growth in GDP for Barbados

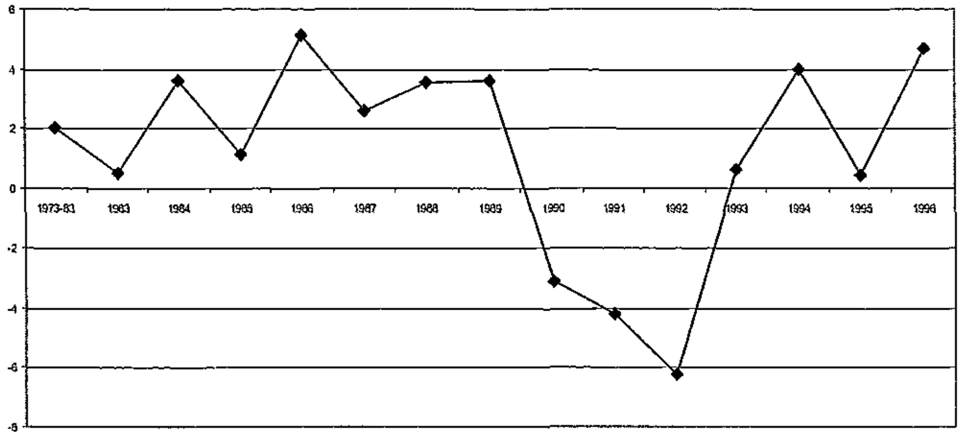


Table 1. Growth in Real GDP

Country	1973-83	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997
Antigua & Barbuda	3,9*	6,9	7,5	8,8	9,7	8,8	7,7	6,3	3,5	4,3	4,3	3,5	3,0	-4,2	5,0	
Barbados	2,0	0,5	3,6	1,1	5,1	2,6	3,5	3,6	-3,1	-4,2	-6,2	0,6	4,0	0,4	4,7	
Belize	3,6	-0,9	1,3	0,6	3,2	11,4	6,8	12,5	8,9	4,6	10,3	3,5	1,6	3,4	1,7	1,4
Dominica	1,3	2,1	5,4	1,7	6,8	6,8	7,4	-1,1	6,3	2,0	2,6	2,2	2,1	1,8	3,2	
Grenada	3,5*	1,2	5,6	4,9	5,5	6	5,3	5,7	5,2	3,2	0,6	-1,3	2,3	2,7	3,0	
Guyana	-1,1*		2,1	1	0,2	0,8	-2,6	-4,9	-3,0	6,0	7,8	8,2	8,5	5,5	7,9	7,0
Jamaica	-1,4*	2,3	-0,9	-4,6	1,7	7,7	2,9	6,8	5,5	0,5	1,4	1,2	1,0	0,5	-1,7	-2,8
Montserrat	3,4*	-4,3	1,8	6	2,8	5,3	9,4	11,5	14,7	-23,7	-1,5	-0,3	0,2			
St. Lucia	4,1*	5,1	9,7	4,5	7,2	6,3	8,6	7,2	7,1	3,1	6,5	1,4	2	4,1	3,7	
St. Kitts & Nevis	3,3*	-1,1	9	5,6	6,2	7,4	9,8	6,7	3,0	3,9	3,0	4,5	3,0	3,9	5,8	
St. Vincent & The Grenadines	3,5	4,3	6,7	9	14,9	1,7	12	9,1	4,4	2,3	7,1	2,3	2,2	4,4	3,3	
Trinidad & Tobago	3,6	1,4	-3,7	-5,9	-1,8	-4,6	4	-0,7	1,5	2,7	-1,7	-1,7	4,0	2,4	3,2	

Sources: CDB Annual Report, World Bank

(*) Data available for 1974-1985

1.2. Definition of "The Barbadian Business Cycle"

One important lesson to be learned from these observations is that economic fluctuations in Barbados <u>mettent en avant</u> a configuration of cyclical movements, which is translated into more or less pronounced periods of decline in the level of GDP during recessionary phases.

Like other Caribbean countries, Barbados is a small island state with tiny internal markets, as well as limited endowments of natural resources and other production factors. As a result, the economy is very export-oriented and extremely dependent on its ties with industrialised countries. Its growth experience over the last four decades has therefore been very much linked, on the one hand, to the potential to export implied by preferential agreements for access to large markets and, on the other hand, to increases in public expenditure funded through its institutional relationships with Europe and North America.

However, due to this high degree of openness, the Barbadian economy is particularly vulnerable to shocks, particularly exogenous shocks such as natural catastrophes, changes in the rules of engagement for accessing European and North American markets, fluctuations in global demand for its exports and varying levels of access to external financing.

Generally speaking, in light of the experience of these last few decades, the volatility of global demand appears to have a direct effect on the economic performance of Caribbean countries. For example, the annual average growth in real GDP for industrialised countries went from 3.5% between 1983 and 1989 to 1.5% in 1990-93. During the same periods, their imports fell from 7% to 3%. For Caribbean countries, this slowdown in the growth of the industrialised countries had negative current account and terms of trade implications, as the prices of their exports declined.

These factors determine the economic fortunes of the countries of the region, resulting in alternating phases of prosperity and recession of irregular duration. Therefore, one may hark back to the classical definition by Burns and Michell (1946) in characterising the Barbadian business cycle. From a purely practical perspective, and in the case of quarterly variables, this definition leads to the following characterisation of the components of a series: the long-term movements in the trend may be more than 32 quarters in duration, while short-term movements may be between 6 and 32 quarters in length for the cycle and less than 6 quarters for the irregular component.

II. Overview of methodological principles

Although trend-cycle decomposition techniques have been used for over two decades, consensus has yet been reached on the best method. Nonetheless, the vast research in this area has helped to clarify the theoretical advantages and limitations of the various techniques, evaluate their performance and predict their results.

The numerous decomposition methods, which can be classified as either non-adaptive filters with fixed parameters, which do not depend on the time series being analysed, and adaptive filters, which, in contrast, are data-specific, with their weights being calculated from the particular time series. Since they are well documented in the literature, it is not necessary to give a detailed exposition of these various decomposition techniques here. Therefore, only the essential points to note about the methods used in this study will be presented.

Non-adaptative Filters

The Hodrick-Prescott (HP) Filter

The HP technique is essentially statistical, in the sense that it isolates the trend and cycle components without explicitly describing them using structural equations (see Appendix 1 for a brief exposition.)

The application of the HP filter involves an assumption as to the value of the λ parameter. It is widely understood these days that the often systematic use of the value 1600 essentially leads to a choice of filter "passe haut", which eliminates all of the components with a periodicity greater than 32 quarters. Thus, the choice implies an a priori assumption as to the duration of cycles. This arbitrary choice of λ and the absence of an econometric estimation process allowing for the choice of value to be made on the basis of the data available are the main weaknesses identified in this method.

The search for satisfactory solutions to these problems have recently mobilised the attention of econometricians. Pedersen (2001) has therefore proposed the optimal value of λ as that which minimises the distorting effect of filtering the time series considered. In his conclusion, he stresses that, "if the cyclical component of seasonally adjusted time series is defined as cycles with a duration less than 32 quarters, we recommend the use of the Hodrick-Prescott filter with a value of the smoothing parameter in the range 1000-1050 when filtering near-integrated time series".

Adaptative filters

3. Trends and cycles of the production by sectors

3.1 Data and stylised facts

The sectoral data used in this paper are estimated quarterly series provided by the Central Bank of Barbados. The first publication of these series was in 1997 (Lewis, 1997). The fact that this is so recent illustrates well the difficulties encountered by some countries, especially developing countries, in presenting quarterly national accounts data.

Indeed, the existence of this type of high-frequency data is a pre-requisite for economic policymaking and financial programming. Bloem, Dippelsman and Maehle (2001) give a telling example: "The recent financial crises taught us that availability of timely key high-frequency data is critical for detecting sources of vulnerability and implementing corrective measures in time".

Given the importance of this type of information, statisticians and econometricians were quick to develop disaggregation techniques, which could provide the necessary data. Since the beginning of the 1960s, a specialist literature has sprung up around the topic, encompassing in particular the work of various American and Anglo-Saxon authors under the generic term "benchmarking". The majority of the methods developed under this rubric seek to generate an estimated quarterly series for a variable from the annual series for the same variable and one or several other quarterly indicators. In general, these methods are classified in two ways: disaggregation methods that reference a statistical modelling technique such as linear stochastic modelling or space-state modelling, or purely numerical methods, encompassing those based on minimum least squares.

Bassie (1958) was the first to propose a way to construct a monthly or quarterly series whose short-term movements faithfully reflected those of an annual reference series. Although this method was used by various economic organisations, notably by the OECD, up until the beginning of the 1980s, its performance has never merited a place among the more robust techniques. On the other hand, the Denton method, proposed in 1971, is often cited as one of the best performing methods and remains in use today by the IMF, which considers it superior to others. The basic concept is the construction of the desired series in such a way that it is proportional to a quarterly indicator. From there, one minimises, using least squares, de la différence en ajustement relative aux nearest quarters subject to the constraints imposed by the annual reference series.

Lewis (1997) uses a sectoral approach for Barbados. The GDP series is built up from estimates of the economic sectors by using available indices of real output and sectoral employment as a proxy of sectoral production when indices are not available.

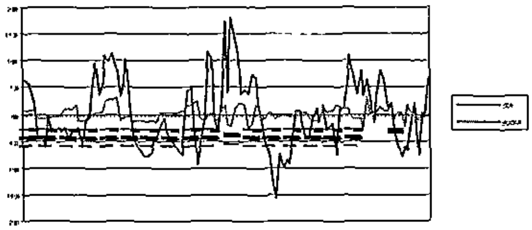
Table 1 : Evolution of the sectoral decomposition of the Barbadian GDP

Sector name	1974		1984		1994		2003	
	value	%	value	%	value	%	value	%
Traded sector								
Sugar	46,97	7,33	42,28	5,44	22,30	2,70	15,61	1,55
Non-sugar Agricultural & Fishing	21,60	3,37	32,93	4,24	31,13	3,77	34,51	3,44
Manufacturing	62,65	9,77	90,33	11,62	78,92	9,55	80,68	8,04
Tourism	64,79	10,10	93,32	12,01	128,97	15,60	154,31	15,37
Non-trades sector								
Mining & quarrying	0,80	0,13	6,72	0,86	5,96	0,72	6,50	0,65
Electricity, Gas & Water	9,50	1,48	20,40	2,62	28,60	3,46	40,85	4,07
Construction	52,83	8,24	50,60	6,51	43,20	5,22	93,21	9,28
Wholesale & retail	118,31	18,45	146,74	18,88	161,14	19,49	193,33	19,26
Government	97,11	15,15	101,58	13,07	113,11	13,68	135,45	13,49
Transportation, Storage & Communications	43,61	6,80	57,00	7,33	68,13	8,24	82,80	8,25
Business and other services	123,01	19,19	135,35	17,41	145,34	17,58	166,71	16,60
Total GDP	641,17	100	777,24	100	826,80	100	1003,97	100

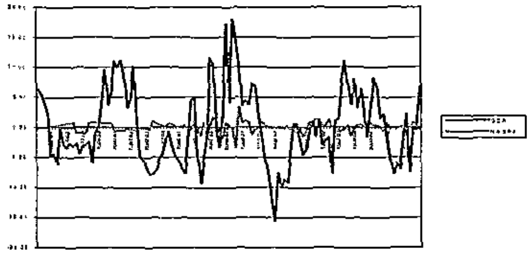
CHARTS OF THE 11 ORGINAL TIME SERIES

Quarterly data

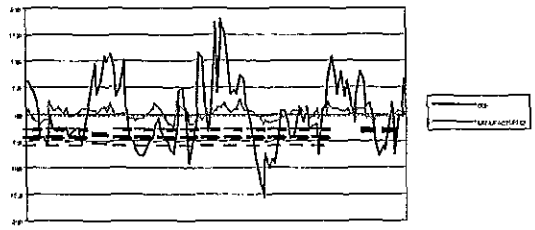
Sugar



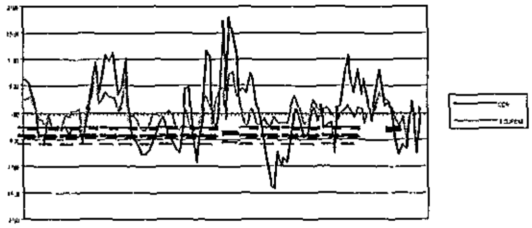
Non-Sugar agricultural & Fishing



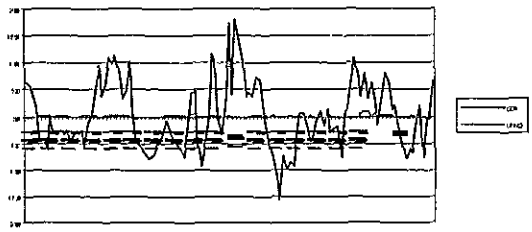
Manufacturing



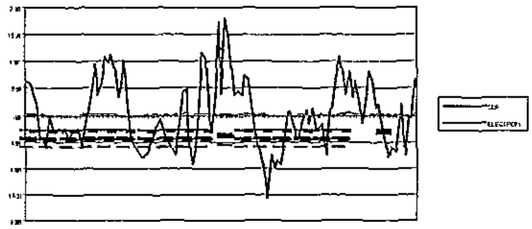
Tourism



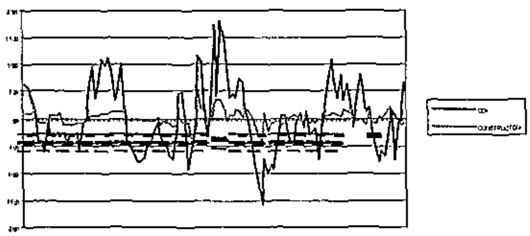
Mining & Quarrying



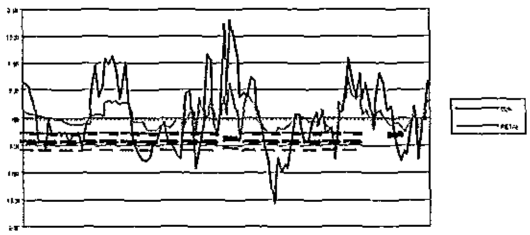
Electricity, Gas & Water



Construction

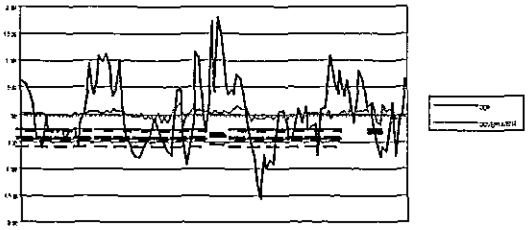


Wholesale & Retail

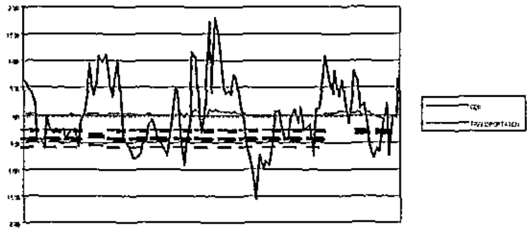


Quarterly data

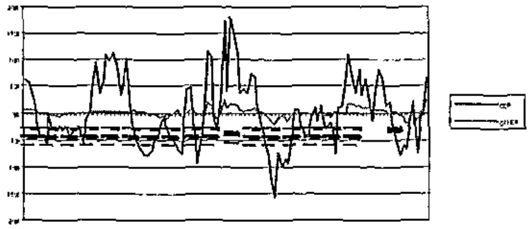
Government



Transportation, storage & communications



Business and other service



3.2 Cyles and sources of growth

The empirical investigations have been carried out on the logarithms of the highest seasonally adjusted series. These cover the period from the first quarter of 1970 to the fourth quarter of 2003, or more than three complete decades, which is more than adequate for characterising the short-to-medium term evolutions observed for Barbados.

It is also important to note that the GDP time series data are different from those used in Craigwell and Maurin (2001). Indeed, this data has been upgraded and updated based on some methodological changes instituted by the Barbados Statistical Service and the Central Bank of Barbados.

WINRATS 4.2 and ??? software were used to carry out the econometric estimations.

Results of the Hodrick-Prescott Filter

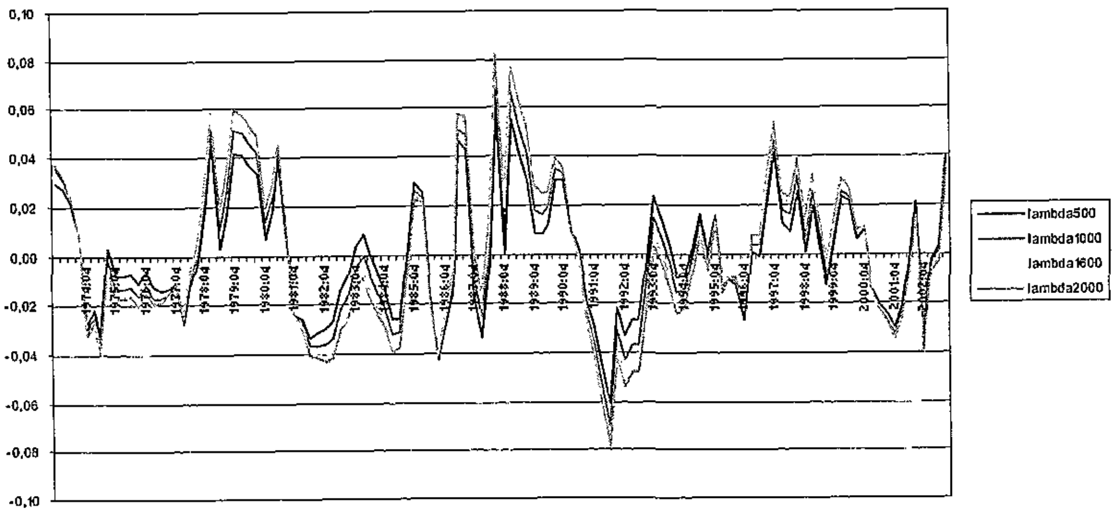
The HP Filter was applied using varying values of λ. Table 2 and Figure 2 below clearly illustrate the following points: ???

However, in the special case of this Barbadian series, the results show that the chronology of the cycle remains almost constant for the different values considered.

Table 2. Summary Statistics of the Cyclical Components of GDP by sectors for Barbados using the Hodrick-Prescott filter with four values for λ

	Standard Deviation	Skewness	Kurtosis	Autocorrelations					
				Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6
λ=250	0,0211	0,1853	-0,4233	0,4722	0,1259	-0,0906	-0,0662	0,0146	0,0139
λ=500	0,0239	0,2477	-0,4162	0,5762	0,2829	0,0820	0,0656	0,0919	0,0536
λ=1000	0,0277	0,3037	-0,3916	0,6761	0,4384	0,2610	0,2149	0,1985	0,1302
λ=1600	0.0307	0.3187	-0.4000	0.7326	0.5284	0.3676	0.3082	0.2714	0.1887
λ=2200	0,0328	0,3188	-0,4165	0,7643	0,5794	0,4273	0,3632	0,3161	0,2262

Figure 2. Cyclical component of Barbadian real GDP for different values of λ



Based on these estimations of the cyclical component of the Barbadian GDP series, and applying the same approach used in Craigwell and Maurin (2001), the following chronology has been established.

Table 3: Peaks and Troughs of Barbados Real GDP Cycle between 1974 and 2003

	Troughs			Peaks		
Date	Amplitude		Time elapsed since last trough (in quarters)	Amplitude		Time elapsed since last peak (in quarters)
	HP	BK		HP	BK	
1974:1						
1975:3	-0,76					
1980:1				1,10		
1983:1	-0,77		30			
1986:1				0,44		20
1986:4	-0,77		15			
1988:4				1,45		11
1992:3	-1,43		23			
1996:1				0,23		29
1997:1	-0,42		18			
1998:1				0,94		8
2003:1	-0,71		24			
2003:4				0,64		19

HERE ARE THE RESULTS FOR THE GDP + THE 11 sectoral times series

Table 3. Summary Statistics of the Cyclical Components of GDP by sector for Barbados
Method: Hodrick-Prescott with λ=1600

	Standard Deviation	Skewness	Kurtosis	Autocorrelations Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6
Total GDP	0.0307	0.3187	0.4000	0.7326	0.5284	0.3676	0.3082	0.2714	0.1887
Sugar	0.1809	-0.7539	2.5474	0.1169	-0.0308	0.0294	-0.0091	-0.0813	-0.1059
N-S Agro	0.1043	-1.0796	5.0430	0.1060	0.0125	-0.0832	-0.0636	-0.0495	-0.1819
Manu	0.0514	-0.1856	-0.0359	0.5385	0.3464	0.0223	-0.1178	-0.1050	-0.0877
Tour	0.0938	0.3471	0.2108	0.5719	0.3471	0.2444	0.1483	0.1312	0.0428
Mining	0.1553	-0.0113	1.5295	0.5499	0.2052	0.0622	-0.0978	-0.0540	0.0869
Elect	0.0332	-0.2249	2.0239	0.5735	0.3562	0.3169	0.0929	0.0548	-0.0154
Construct	0.1000	-1.1366	6.0246	0.4627	0.2536	0.1326	0.0373	0.1190	0.1725
Retail	0.0512	0.3329	0.4767	0.5763	0.4057	0.3079	0.1702	0.0970	0.0636
Gov	0.0256	-0.4655	2.0224	0.2731	0.2250	0.0859	-0.0120	0.1896	0.1322
Transcom	0.0195	0.0789	0.4368	0.5564	0.2919	0.1826	0.2262	0.2071	0.0865
Other	0.0207	0.1759	0.9917	0.6169	0.3684	0.2111	0.2454	0.1532	0.0116

Table 3: Correlations between the Cyclical Components of total GDP and production by sectors of Barbados
Method : Hodrick-Prescott with four values for λ

Lag	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
Total GDP													
Sugar													
Nagrf													
Manu													
Tour													
Mining													
Elect													
Construc													
Retail													
Gov													
Transcom													
other													

Contributions of the sectoral cycles to the standard deviation of the GDP cycle

Given that overall GDP is obtained by aggregating the value-added of the various sectors, it is possible to calculate the contribution of the cyclical component of each sector to the variance in the cyclical component of GDP. If we take z_c to be the cyclical component of sector z, filtered in levels, the contribution of the cycle of this series to the variance of the GDP cycle is equal to the ratio:

$$\frac{\mathrm{cov}(GDP\_c, z\_c)}{\mathrm{var}(GDP\_c)}$$

This contribution can also be decomposed into the product of the correlation between the two cyclical components and the ratio of their standard deviations using the following identity:

$$contribution(z) = \frac{\mathrm{cov}(GDP\_c, z\_c)}{\mathrm{var}(GDP\_c)} = correlation(GDP\_c, z\_c) \times \frac{\sigma(z\_c)}{\sigma(GDP\_c)}$$

Thus, the contribution of the cycle of an element of the GDP cycle to the variance of the GDP cycle is greater than the degree of correlation between the cyclical components (this element fluctuates in the same direction as GDP) and greater than the cycle of this element.

It can be shown that the summation of the different contributions is equal to 1. In effect:

If $GDP = z_1 + z_2 + ... + z_k$, with, in the case of Barbados, $z_1, z_2,..., z_k$ representing the 11 productive sectors, respectively, then $GDP\_c = z_1\_c + z_2\_c + ... + z_k\_c$, where GDP_c represents the cyclical component of GDP, z_1_c, that of sector z_1...

Therefore:

$$\text{var}(GDP\_c) = \text{cov}(GDP\_c, z_1\_c) + \text{cov}(GDP\_c, z_2\_c) + ... + \text{cov}(GDP\_c, z_k\_c)$$

It should be noted that these relationships do not make sense unless the series are filtered in levels and not in logs. The cyclical components are expressed in the same units as their parent series.

Contributions of the sectoral cycles to the standard deviation of the GDP cycle

Sector name	Corrélation avec le cycle du GDP	Ecart-type/écart-type du GDP	Contribution
Traded sector			
Sugar	028	0,19	5,37
Non-sugar agricultural & fishing	-0,01	0,12	-0,12
Manufacturing	0,54	0,7	9,03
Tourism	0,73	0,38	28,17
Non-trades sector			
Mining & quarrying	0,18	0,03	0,56
Electricity, Gas & Water	0,18	0,03	1,19
Construction	0,45	0,20	13,06
Wholesale & retail	0,64	0,34	25,04
Government	0,74	0,12	5,29
Transportation, storage & communications	0,63	0,05	3,39
Business and other services	0,73	0,12	9,02

The values calculated for these different correlations and contributions to the GDP cycle appear to be in keeping with the dynamic of the Barbadian economy over the last three decades. Firstly, *Tourism* and *Wholesale & Retail* contribute the most to the cycle in the amounts of 28.17% and 25.04%, respectively. These contributions are relatively large because they are significantly correlated with the GDP cycles and also because the cycle itself is very large. *Construction, Business & Other Services* and *Manufacturing* make medium contributions, whereas *Sugar, Mining & Quarrying, Electricity, Gas & Water* and *Transportation, Storage & Communications* make a very modest contribution to the cycle (less than 6%).

These positive correlations also emphasise the fact that only the *Non-Sugar Agriculture & Fishing* sector contributes negatively to the variance in the GDP cycle and therefore appears to be moving counter-cyclically.

IV. Trend and cyclical interactions of GDP with some variables : prices, employment, interest rate

Appendix 1: The Hodrick-Prescott (HP) filter

Formally, the method consists of minimising the weighted sum of the two components of the following function:

$$\underset{\{T_t\}_{t=1}^{n}}{Min} \sum_{1}^{n} (T_t - Y_t)^2 + \lambda \sum_{3}^{n} [\Delta T_t - \Delta T_{t-1}]^2$$

The first term measures the degree of approximation of the trend, T_t, to the series, Y_t, while the second term, the degree of variability in the trend (sum of squares of the variations in the growth rate of the trend). The parameter λ represents the weight assigned to the second component relative to the first, and signifies an opportunity cost linked to the introduction of fluctuations in the trend. It plays a crucial role in the decomposition, as the higher the value of λ, the more important the trend's smoothing component (component 2), relative to its degree of approximation to the raw series (component 1) and, therefore, the greater the importance of the cyclical component.

Thus, for $\lambda = \infty$, the problem is reduced to minimising $\sum_{3}^{n} [\Delta T_t - \Delta T_{t-1}]^2$, thereby obtaining $\Delta T_t = C^{te}$, i.e., a constant growth rate for the trend, which corresponds to a linear trend. On the contrary, for λ =0, the problem involves minimising $\sum_{1}^{n} (T_t - Y_t)^2$, from which we obtain $Y_t = T_t$ and $C_t = 0$ for all points in time, which would indicate that the trend series matches the raw series exactly.

The flexibility of the HP technique, allows λ to be chosen to permit calibration of the duration of the cycle. For quarterly series, the most commonly used value of λ is 1600. Since this value eliminates low frequencies, thus filtering out the cyclical elements with the longest durations (in this study, greater than 32 quarters). The cycles thus identified have an average duration of 4 to 6 years.

The arbitrary selection of the parameter λ may be justified based on the ratio of the variances of the two components. In other words, in the very special case where C_t and $\Delta^2 T_t$ are independent white noise processes with respective variances σ_1^2 and σ_2^2, the minimisation problem is reduced to calculating the conditional expected value of T_t given Y_t (i.e., the best possible estimate of the trend given that the raw series is unknown), since λ is equal to the ratio of the variances $\frac{\sigma_1^2}{\sigma_2^2}$. Within this framework, and using deviations from the trend of 5% per quarter and changes in the growth rate of the trend of 5% per quarter, as a proxy for these two variances, the value obtained is $\lambda = \frac{5^2}{\left(\frac{1}{8}\right)^2} = 1600$.

Naturally, this justification remains heuristic, mainly because in practice the cycle under examination is not a white noise process. The choice of λ , therefore, remains arbitrary, which renders the decomposition less than robust. Moreover, Hervet and Jaeger (1993) show that a perfunctory utilisation of this filter could ultimately lead to the false identification of cycles in a series stripped of its cyclical component (a random walk, for example), due to a Slutsky-Yule-type effect.

Appendix 2: The Beveridge Nelson decomposition

Beveridge-Nelson

Based on an ARIMA model of the series under examination, the Beveridge-Nelson approach involves decomposing an I(1) series into the sum of its permanent and transitory components.

Using an I(1) series (Y_t), which satisfies the following ARIMA formulation:

$$(1 - L)\varphi(L)Y_t = b + \theta(l)\zeta_t$$

Where L is a lagged operator, ζ_t a white noise process, and the moduli of the square roots of the polynomials φ and θ are strictly greater than 1. In this case, ζ_t is the innovation of $(1-L)Y_t$ and the preceding equation may alternatively be written in the following form:

$$(1-L)Y_t = \beta + A(L)\zeta_t$$

where

$$A(L)\zeta_t = \sum_{k=0}^{\infty} \alpha_k \zeta_{t-k} \quad \text{and} \quad \alpha_0 = 1$$

This second equation is known as the ***Wold Decomposition*** of (Y_t).

One could also write:

$$(1-L)Y_t = \beta + A(1)\zeta_t + [A(L) - A(1)]\zeta_t$$

or, alternatively, given that:

$$\alpha = A(1) = \sum_{k=0}^{\infty} \alpha_k$$

one would obtain:

$$(1 - L)Y_t = \beta + \alpha\zeta_t + (1 - L)a(l)\zeta_t$$

This decomposition of the growth rate of the series allows the calculation of the level of the series in terms of $t+h$ using the summation:

$$Y_{t+h} = Y_t + \beta h + \alpha \sum_{k=1}^{h} \zeta_{t+k} + a(L)\zeta_{t+h} - a(L)\zeta_t$$

As ζ_t is, by its construction, innovation at time t, the best estimate for horizon h given all the observations up to time t may therefore be written:

$$E_t(Y_{t+h}) = Y_t + \beta h + \sum_{k \geq h} a_k \zeta_{t+h-k} - a(L)\zeta_t$$

Note that:

$$a(L) = \sum_{k=0}^{\infty} a_k L^k$$

If h is large, the term $\sum_{k \geq h} a_k \zeta_{t+h-k}$ tends towards zero. La suite des prévisions d'horizon H se situe donc approximativement sur la droite de pente β, passant par le point $T_t = Y_t - a(L)\zeta_t$.

Beveridge and Nelson therefore define T_t as the value of the trend at time t. This value is therefore the value the series would have taken at time t if it were situated on its long-term growth path.

The trend T_t is a random walk, given that:

$$(1-L)T_t = (1-L)Y_t - (1-L)a(L)\zeta_t$$

that is:

$$(1-L)T_t = \beta + A(1)\zeta_t.$$

The deviation C_t between the series and its trend can be written: $C_t = a(L)\zeta_t$. It is stationary and autocorrelated. Thus, the series has been decomposed into the sum of a random walk, i.e. a non-stationary series, whose first differences are not autocorrelated (the trend), and a stationary, autocorrelated series (the cycle). Again, the path of the cycle will fluctuate around a zero mean, without, *a priori*, displaying periodicity.

This method of modelling the trend and the cycle depends on an assumption that the innovations ζ_t operating in the two components are identical to a close multiplicative factor and, as a result, the two components are perfectly correlated.

[[[[cf insert papers to compare]]]]

Appendix 3 : Models using Unobserved Components

The Beveridge-Nelson (   ) approach assumes that the cycle and the trend are perfectly correlated. Therefore, a shock to the trend is accompanied by a shock to the cycle. The approach using unobserved components outlined below (the most frequently used in the literature), however, is based on a diametrically - opposite hypothesis, which assumes that the cycle and the trend possess independent characteristics.

The different types of models

In order to decompose the cycle and the trend, one must restrict the dynamic of each component. The most commonly used hypothesis assumes, as in the Beveridge-Nelson decomposition, that the autocorrelated portion of the dynamic is all contained in the cycle and that the movements in the trend are independent.

One class of models is obtained by imposing the limitation that the trend T_t is a random walk with the formulation:

$$T_t = T_{t-1} + \beta + \eta_t \quad , V\eta_t = \sigma_\eta^2 .$$

In another approach, the slope of the trend is a variable that follows a random walk. The model describing the trend is therefore:

$$T_t = T_{t-1} + \beta_{t-1} + \eta_t$$
$$\beta_t = \beta_{t-1} + \zeta_t$$

with $V\eta_t = \sigma_\eta^2$ and $V\zeta_t = \sigma_\zeta^2$, where (η_t) and (ζ_t) are independent white noise variables.

Here, the deterministic component of the first model is progressively broken down as follows: η_t represents shocks to the trend in levels, while ζ_t represents shocks to the slope of the trend. The greater the variance of the innovations in the slope, the less smooth the trend becomes. In this model, the trend is therefore integrated of the second order. This hypothesis implies that the series to be decomposed is (2), which is not a popular assumption in the current literature.

All of the most common specifications can be obtained by imposing the following constraints on the second model:

- If $\sigma_\zeta^2 = \sigma_\eta^2 = 0$, then $\beta_t = \beta$ for all t, even though the trend becomes deterministic;
- If $\sigma_\eta^2 \neq 0$, then $\beta_t = \beta$ for all t. In this case, the trend is a random walk with a model that is identical to the previous one;
- $\sigma_\eta^2 = 0$, then $\beta_t = \beta_{t-1} + \zeta_t$, and the trend is the sum of a deterministic trend and an I(2) process.

The next stage involves specifying the cycle's dynamics. It is generally assumed that the cycle is an ARMA process, independent of the shocks that affect the trend. The square roots of the autoregressive polynomial will therefore play a determining role, such that the closer the moduli of these square roots (assumed by definition to be outside the "unit circle") to one, the more persistent the autocorrelation.

In the cyclical model proposed by Harvey, an auxiliary process C_t^* intervenes, such that the vector (C_t, C_t^*)' conforms to the following dynamics:

$$\begin{pmatrix} C_t \\ C_t^* \end{pmatrix} = \begin{pmatrix} \rho \cos \omega & \rho \sin \omega \\ -\rho \sin \omega & \rho \cos \omega \end{pmatrix} \begin{pmatrix} C_{t-1} \\ C_{t-1}^* \end{pmatrix} + \begin{pmatrix} k_t \\ k_t^* \end{pmatrix}$$

k_t and k_t^* being independent noises with identical variances.

It can be shown that the formulation of C_t thus obtained is equivalent to a ARMA (2,1) process, in which the coefficients verify a certain number of constraints. The stationary process solving this system of equations is identified as "quasi-cyclical". In fact, the deterministic framework obtained by nullifying these innovations models a phenomenon of amortised oscillations ($\rho < 1$), the duration τ of the oscillations being equal to. This property of the framework is offset by the « quasi-cyclical » nature of the stochastic process: a cycle which is positive at time t tends to be positive at time $t+\tau$, and the autocorrelogram of the cycle shows peaks decreasing in size in line with lags of τ, 2τ,... This property is reflected in a peak ω with a process spectral density C_t.

Estimation Methods

The process of decomposing the trend and cycle based on these models are more difficult to use than the Beveridge-Nelson method. It requires the use of the Kalman filter for both estimating the parameters of the model and extracting the trend and cyclical components.

Firstly, the accuracy of the model is tested and maximised using an optimisation algorithm. Once the parameters of the model have been estimated, the trend and the cycle are extracted (see table). Due to the structure of the model, the fluctuations in the series Y_t do not allow for an unambiguous identification of the two components. The optimal estimate is calculated at each point in time using the available information and this estimate is translated using a somewhat large confidence interval. This constitutes a key departure from the Beveridge-Nelson method, where, once the ARIMA model has been specified, the trend and the cycle are perfectly defined – the only potential source of error being the estimation of the parameters.

<u>Encadré : La méthode du filtre de Kalman</u>

Consider a model using unobserved variables of the following form:

(1) $Y_t = Z_t \alpha_t + d_t + e_t$

(2) $\alpha_t = A_t \alpha_{t-1} + c_t + R_t \eta_t$

with

$Ee_t = 0$, $Ve_t = H_t$

$E\eta_t = 0$, $V\eta_t = Q_t$.

The first equation is called the measurement equation and the second is referred to as the transition equation. The unobserved coefficient α_t is called the state vector.

The Z_t, A_t, d_t, c_t, R_t, H_t and Q_t matrices may vary over time but in a deterministic fashion (this assumption may later be relaxed under certain conditions). These matrices generally depend on unknown parameters, which will be estimated using the maximum likelihood method.

The Harvey model is written:
$Y_t = Z_t \alpha_t + e_t$
with
$Z_t = (1 \quad 0 \quad 1 \quad 0)$, $\alpha_t = (T_t \quad \beta_t \quad C_t \quad C_t^*)'$

and α_t verifies the following equation :

$$\alpha_t = \begin{pmatrix} T_t \\ \beta_t \\ C_t \\ C_t^* \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \rho\cos\omega & \rho\sin\omega \\ 0 & 0 & -\rho\sin\omega & \rho\cos\omega \end{pmatrix} \begin{pmatrix} T_{t-1} \\ \beta_{t-1} \\ C_{t-1} \\ C_{t-1}^* \end{pmatrix} + \begin{pmatrix} \eta_t \\ \zeta_t \\ K_t \\ K_t^* \end{pmatrix}$$

<u>The Filtering Process</u>

For a given value of the vector of parameters, the Kalman filter consists of calculating iteratively the best estimator of the state vector for each time-period *t*, given the information available up to time (*t-1*) or up to time t (i.e. the best linear estimator for α_t given this information, which coincides with the conditional expectation assuming normality.

More precisely, if one takes $\alpha_{t-1} = EL(\alpha_{t-1} / I_{t-1})$ to be the best linear estimator of α_{t-1} given $I_{t-1} = \{Y_{t-1}, Y_{t-2}, ...\}$, $a_{t/t-1} = EL(\alpha_t / I_{t-1})$ the best linear estimator of α_t given I_{t-1}, and given that

$$P_{t-1} = E\left[(\alpha_{t-1} - a_{t-1})(\alpha_{t-1} - a_{t-1})'\right]$$

and

$$P_{t/t-1} = E\left[(\alpha_{t-1} - a_{t/t-1})(\alpha_{t-1} - a_{t/t-1})'\right]$$

are the variance-covariance matrices of the corresponding forecasting errors, one can calculate for each time $a_t, a_{t/t-1}, P_{t-1}, P_{t/t-1}$ using the following iterative formulae:

(3.1) $a_{t/t-1} = A_t a_{t-1} + c_t$

(3.2) $P_{t/t-1} = A_t P_{t-1} A_t' + R_t Q_t R_t'$

(4.1) $a_t = a_{t/t-1} + P_{t/t-1} Z_t' F_t^{-1} (Y_t - Z_t a_{t/t-1} - d_t)$

(4.2) $P_t = P_{t/t-1} - P_{t/t-1} Z_t' F_t^{-1} Z_t P_{t/t-1}$

(4.3) $F_t = Z_t P_{t/t-1} Z_t' + H_t$

Equation (4.1) is an updating equation: it essentially gives the best estimation of α_t at time *t* (that is, once Y_t is known) given the best prediction of α_t at time (*t-1*).

Estimation of the Vector of Parameters by Maximum Likelihood

Given observations $Y_1,...,Y_T$, $l(Y_1,..., Y_T, \psi) = \prod_{t=1}^{T} f(Y_t / I_{t-1})$, using $f(Y_t/I_{t-1})$ to represent the conditional density of Y_t given $(Y_1,...,Y_{t-1})$, and using ψ to represent the vector of parameters.

Assuming that the errors are normally distributed, the conditional density of Y_t given I_{t-1} is the normal distribution $N(Z_t a_{t/t-1} + d_t, F_t)$. Thus, the model likelihood is expressed as a function of the progressive estimations produced via the previously described filtration process, which occurs for each value of the vector of parameters ψ.

For each value of ψ the Kalman Filter is applied to calculate a_t and $a_{t/t-1}$ at each point in time and thereby determine the likelihood.

ψ is then estimated using the Maximum Likelihood approach, with the help of an optimisation algorithm (since the solution cannot be calculated arithmetically)

The Smoothing Process

Once the vector of parameters is estimated, the filter is applied once more, using the estimated value of the vector to obtain for each time-period the best estimator of the state vector α_t, which corresponds to that particular value. However, it is clear that the best estimator of α_t cannot be obtained in this way. In fact, one obtains a better estimation of the state vector by calculating for t = T to 1:

$a_{t/T} = EL(\alpha_t / I_T)$ or $I_T = \{Y_1,..., Y_T\}$.

once again using iterative methods, as well as the following formulae:

$$a_{t/T} = a_t + P_t^*(a_{t+1/T} - a_{t+1/t})$$

with

$$P_t^* = P_t T_{t+1}' P_{t+1/t}^{-1}$$

and the variance-covariance matrix of the error term is:

$$P_{t/T} = P_t + P_t^*(P_{t+1/T} - P_{t+1/t})P_t^{*'} .$$

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