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An Assessment of Volatility Transmission in the Jamaican Financial System

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Abstract

This paper applies the GARCH-BEKK procedure to the returns from the Jamaican bond, foreign exchange and stock markets in order to estimate the magnitude of the common market and cross-market volatility transmission. The behaviour of these spillover effects over a specified period is then assessed. In particular, the paper employs a simple VAR procedure that uses the variance series of the three market returns derived from the GARCH-BEKK model as the endogenous variables. The results of the model suggest that within the Jamaican financial system, there are generally high levels of common market volatility transmission, relative to cross-market volatility transmission. Of the three markets, the foreign exchange market exhibits the most pronounced common market volatility transmission, followed by the stock market. Strong common market transmission in these two markets, relative to that of the bond market reflects the uncertainty momentum that often characterises these risky markets. The strongest cross-market effects occur from the bond market to the foreign exchange and stock markets. Additionally, the findings of the paper suggest a negligible impact of Government and Central Bank bond maturities on volatility transmission within and across markets. This is interpreted as evidence of the successful management of liquidity through open market operations.

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[1] The views expressed are those of the author and do not necessarily reflect those of the Bank of Jamaica.

1. Introduction

Asset prices are generally influenced by the portfolio decisions of investors who actively participate in more than one financial market. These decisions are usually influenced by a continuous flow of information that often results in price volatility spillovers within and across markets[2]. Market efficiency proponents generally attribute the spillover effects to inefficiencies in market structures, particularly in the dissemination of relevant information to market participants. These spillovers could reflect a failure of market efficiency as it should not be possible to predict returns or volatility in one market using past information. However, if news about fundamentals were serially correlated, then the existence of spillovers would not necessarily imply a failure of market efficiency[3].

Close examination of the nature of volatility transmission is important in aiding the effectiveness of monetary policy and in addressing financial stability issues. With regard to monetary policy, it is critical to understand the manner in which shocks are propagated across markets in order to determine the persistence of these innovations and the magnitude of their effects over time.[4] The extent to which volatility is transmitted across markets could result in a large shock in one market destabilizing another market. The ability of policy-makers to gauge the depth and duration of the impact of cross-market and common market shocks can aid the implementation of timely and effective monetary policy. The understanding of the various market price interrelationships is also very important to financial stability. The complexity of these interrelationships represents a potential source of systemic financial instability. To this extent, a comprehension of the intricate market volatility linkages facilitates the implementation of effective mechanisms that allow or encourage entities to hedge against the market risks emanating from shocks that persist within a financial market and those that are propagated across markets.

An explanation of the source of volatility spillovers is offered by modern portfolio theory. Beginning with Markowitz (1952), this theory established the importance of

[2] In this paper, the term "volatility spillover" represents both the common-market case, in which historical volatility in a particular market impacts the current volatility in the same market, as well as the cross-market case, which describes the propagation of shocks from one financial market to another.
[3] See Ebrahim (2000).
[4] Ibid.

finding the optimal balance between portfolio risk and return in the determination of investor demand for a financial asset. Within this framework, the portfolio return reflects the weighted average of the returns from the various assets included in the portfolio, while total portfolio risk is determined by the volatility of the return on each asset group and the joint volatility between the return on all the paired combinations of assets in the portfolio.

Fleming, Kirby and Ostdiek (FKO) provide a useful theoretical explanation for price volatility behaviour[5]. Using a simple model of speculative trading, they employ mean-variance portfolio optimization, as proposed by Markowitz (1952), to derive a theoretical relationship between the demand for asset "futures" and the risk and return of the underlying assets. This relationship provides an implied link between the demand for financial assets and the market return volatilities. In a dynamic setting, the asset returns volatilities have an impact on the demand for the asset, which could cause episodes of common market and cross-market volatility in subsequent periods. Common market volatility arises from investor uncertainty induced from the initial shock to the return of an asset. In explaining the case of cross-market volatility spillovers, Fleming, Kirby and Ostdiek (1996) assert that, as a portfolio manager considers the correlation between different market returns, he will take a position in one market in order to hedge his speculative position in another. In addition to the hedging channel, the model also indicates that cross-market volatility spillovers may generally occur where an information event that alters the expectation about returns in one market will influence demand and trading in another market.

This paper applies the multivariate form of the GARCH procedure[6] to the returns from the Jamaican private bond, foreign exchange and stock markets using the framework proposed by FKO. This empirical model will be used to estimate the coefficients reflecting the extent of common market and cross-market volatility spillovers. Importantly, the influence of changes in market liquidity, in terms of Government and

[5] See Fleming, Kirby and Ostdiek (1997).
[6] This procedure was established by Baba, Engle, Kraft, and Kroner (BEKK) in 1991.

Central Bank bond maturities may need to be explicitly accounted for when computing volatility spillovers. Accordingly, the GARCH-BEKK procedure is carried out with and without the inclusion of liquidity effects in the model, so as to gauge the impact of Jamaica Dollar liquidity on the asset return volatility linkages.[7] In addition, the paper utilizes the estimated variance series as inputs in a simple vector autoregressive (VAR) model to produce 10-day volatility impulse responses. This application details the extent to which the variance of the asset return in a particular market is influenced by the lagged variances of the returns in the same market and the other two markets.

The remainder of the paper is organized in the following manner. Section 2 presents a trading model that provides some intuition for volatility transmissions. Section 3 provides a brief literature review on some of the applications of autoregressive and the generalized auto regressive time series models. Section 4 outlines the specification of the multivariate BEKK model that is employed and data is described in section 5. The estimation results and findings are discussed in section 6. The final section presents the conclusion and policy recommendations.

2. Theoretical Motivation

Following on work done by Tauchen and Pitts (1983), FKO utilized the mean-variance optimisation framework to construct a trading model for financial asset futures. The model assumes that the economy contains a large number of active speculators, who trade with one another because they differ in their expectation about the future and in their need to transfer risks through market transactions. At the start of a trading round, all the financial markets are in equilibrium. This paper adapts the framework of FKO to account for the absence of a formal futures market in Jamaica.

When new information arrives, traders revise their demand for a particular financial instrument and the information event generates a round of trading that continues until the

[7] Although many volatility models concentrate on utilizing historical uncertainty measures to estimate conditional volatility, studies such as that done by Hamilton and Lin (1996) show that certain exogenous variables affect volatility. In this study the effect of domestic dollar liquidity is considered.

market price has reached a new equilibrium. Formally, let S_t^a be the underlying price of asset a at time t and $E\left[S_T^a\right]$ be the expected price of the asset a at time t (where $t < T$) that the investor expects to receive when the asset is sold at time T. A speculator, who takes a long position at time t, expects to earn a profit of $\pi_{t,T}^a$ at time T, where the expectations are conditioned on all available information. The expected profit for the speculator, given his information set, I_t, is given as:

$$E\left[\pi_{t,T}^a / I_t\right] \equiv E\left[S_T^a / I_t\right] - S_t^a \qquad (1)$$
$$t < T$$

Given the above conditions, the portfolio mean-variance optimization (MVO) theory may be used to derive the demand for the asset. The theory assumes that the trader maximizes his expected profit subject to a variance constraint. The standard results derived from this theoretical framework suggest that:

$$Q^a = \frac{E\left[\pi_{t,T}^a / I_t\right]}{2\alpha\sigma^2} \qquad (2)$$

where Q^a is the quantity demanded for asset a, α is the trader's risk aversion coefficient and σ^2 is the variance of the expected return. The impact of the asset variance on the demand in equation (2) highlights a channel through which asset return volatility in one period, by influencing demand and the expectations of the speculative trader, may impact the future volatility of asset returns.

The volatility-trading model may be generalized to allow speculators to trade in more than one asset market, market b and market s, for example[8]. In this case, the demand function exhibits cross-market dependencies. Assuming that the two assets are traded, β denotes the slope coefficient in the linear regression of the expected profit, $\pi_{t,T}^s$, for

[8] The two-asset case is a demonstration of what would occur if there were more than one market for financial assets.

market s, on the expected profit, $\pi^{b}_{t,T}$, for market b. Further, $\sigma^2_{s/b}$ represents the variance of the regression error. Similarly, β_b and $\sigma^2_{b/s}$ denote the slope coefficient and the variance for the regression of the expected value of $\pi^{b}_{t,T}$ and $\pi^{s}_{t,T}$, respectively. For these conditions, the MVO theory derives the following demand functions:

$$Q_s = \frac{E[\pi^{s}_{t,T} / I_t]}{2\alpha\sigma^2_{s/b}} - \frac{E[\pi^{b}_{t,T} / I_t]}{2\alpha\sigma^2_{b/s}} \beta_b \tag{3}$$

and

$$Q_b = \frac{E[\pi^{b}_{t,T} / I_t]}{2\alpha\sigma^2_{b/s}} - \frac{E[\pi^{s}_{t,T} / I_t]}{2\alpha\sigma^2_{s/b}} \beta_s \tag{4}$$

The demand functions, Q_s and Q_b denote the number of assets s and b, respectively, which are demanded by the speculative trader who is betting on the outcome of future movements in the underlying asset prices in order to maximize his overall portfolio returns. The model shows that in the two-market case, the trader's demand for asset s is a function of $\sigma^2_{s/b}$, so the size of his position in this asset depends on the cross variance between the expected profits on asset s and asset b. This cross-variance term captures a volatility impact of one asset, on the demand of another and thus the future volatility of the other asset in the portfolio[9].

The demand functions, depicted by equations (3) and (4), suggest that the trader's demand for an asset, in the two market case, is a function of the expected profits and the cross variances $\sigma^2_{s/b}$ and $\sigma^2_{b/s}$. FKO explain that the cross variances may result from a hedging channel. They note that if the expected profit for one financial asset is zero, the demand could still be non-zero. For example, the model shows that even if the expected profit for asset b is zero, once the investor believes that there is a negative correlation between the two markets, there will still be a positive demand for asset b equal to

[9] The volatility impact will eventually feed into investor expectations and, thus, future asset demand volatility.

$-\beta_S Q_S$[10]. This demand occurs because there are hedging benefits in holding the asset to reduce the overall portfolio risk as long as the two assets have a negative correlation. With the hedge in place, the risk of a long position in the asset is $\sigma^2_{s/b}$. This risk is generally less than the risk of an unhedged position in asset s, so the trader's demand for asset s is increased by the availability of asset b. Where there is news pertaining to the market for asset s, due to hedging, it will lead to an adjustment in the demand for asset s and also the demand for the substitute asset b. Changes in the spot prices of the assets due to hedging would be immediately reflected in the expected future spot price through the no-arbitrage pricing relation. This is the hedging channel through which information in one market may spill over into another market. Equations (3) and (4) also provide a broader interpretation of cross-market volatility spillovers. The fact that the demand for one asset is also dependent on the variance of all the assets in the portfolio, the model implies that an information event that alters the expectation about returns in one market will influence demand and trading in another market. This is a general explanation for cross-market volatility spillovers.

3. Review of Empirical Literature on Asset Price Volatility Spillovers

It has been widely observed that financial time series tend to exhibit volatility clusters. Mandelbrot (1963) reported evidence that large changes in the price of an asset are often followed by other large changes and, similarly, small changes are often followed by small changes[11]. In order to capture these time-varying volatilities, the autoregressive conditional heteroskedasticity model (ARCH) of Engle (1982) and the generalized extension of this model (GARCH) by Bollerslev (1986) have been used extensively. In the model, the variance in the return series is modelled as a function of past variances and past errors that are derived from the regression of the mean return series on its laggs. Maximum Likelihood Estimations are then used to compute the coefficients of the model. It has been found that ARCH and GARCH models provide good in-sample parameter estimates and, when the appropriate volatility measure is used, reliable out-of-sample

[10] Negative correlation of returns gives a negative β_s.

[11] Longmore and Robinson (2004) documented similar findings for the Jamaican foreign exchange market.

volatility forecasts. Among these models, there has been, particularly, much support for the GARCH (1,1) model[12]. In an extensive application of 330 different volatility models to daily US/ DM exchange rates and the daily stock price of IBM, Hansen and Lunde (2001) conducted out-of-sample comparisons that revealed that none of these models provide a significantly better forecast than the GARCH (1,1) model.

A number of univariate GARCH models have been used to measure the extent of volatility spillovers from one financial asset return series to another. Kim and Langrin (1998), for example, used an asymmetric GARCH to model the conditional mean and variance of stock price returns in Jamaica and Trinidad and Tobago with spillover effects from the US stock market[13]. This study revealed the existence of spillover effects from a major US stock market to the markets in Jamaica and Trinidad and Tobago. In a more recent application of the univariate GARCH model, Bala and Premaratne (2002) found evidence of volatility spillovers between the Singapore Stock Market and the markets of USA, Japan and Hong Kong. In another study involving a combination of developed financial markets Kaltenhaeuser (2003) found evidence of stock market volatility spillovers between the markets in the euro area, USA, and Japan.

Although the univariate GARCH approach has a history of success in capturing the effects of volatility spillovers, it has shortcomings. Based on the fact that financial volatilities move together over time across assets and markets, the general view is that Multivariate GARCH models provide a more accurate description of financial market dynamics[14]. One of the earliest rigorous attempts in this category was the VECH model of Bollerslev, Engle, and Wooldridge (1988). This approach extended the basic model of Engle and Bollerslev by using the simultaneous equation form of the original model. The VECH model proved to be a cumbersome approach, as a large number of coefficients had to be estimated, thus utilizing relatively small degrees of freedom in the estimation process. To resolve this estimation problem, Bollerslev (1990) introduced the Constant

[12] See Hansen (2001).
[13] In this study, the authors accounted for the leverage effects that usually cause stock prices to respond more intensively to negative news, relative to positive news.
[14] See Bauwens et al (2004).

Conditional Correlation (CCC) model. This model simplified the estimation of the multivariate GARCH coefficients by imposing restrictions on the variance-covariance matrix derived from the system of simultaneous equations. Although the CCC model was a useful improvement over the VECH, there were apparent drawbacks. Firstly, the major assumption of constant correlations between the different variables in the system of equations was thought to be unrealistic[15]. Furthermore, the model did not ensure that the estimated variance-covariance matrix was positive definite. This condition is necessary to ensure existence of the solution to the system of equations.

In order to avoid the imposition of unrealistic assumptions on the variance-covariance matrix and to circumvent the problem of non-positive definiteness, Engle and Kroner (1995) proposed the BEKK model -named after Baba, Engle, Kraft, and Kroner. This model uses a quadratic form of the parameterization of the original system of equations to ensure the positive definiteness of the variance-covariance matrix without significantly changing the information content of the system of equations[16]. Ebrahim (2000) applied the trivariate BEKK to study price and volatility spillovers between the foreign exchange and associated money markets for three different countries, relative to the USA. In a more recent study, Longmore and Robinson (2004) applied the BEKK model in forecasting foreign exchange market returns in Jamaica.

4. Empirical Model

The trivariate representation of the BEKK model is adopted in this paper to examine the volatilities and pair-wise volatility linkages between the Jamaican stock, bond and foreign exchange markets. In this application, the following BEKK form will be used to model the asset returns and returns volatility of the bond market, foreign exchange market and stock market, labelled as assets 1, 2 and 3, respectively.

[15] For example, Login and Solink (1995), and Karolyi and Stulz (1996) found evidences of time–varying conditional correlations between international equity markets.
[16] There are eleven parameters to be estimated in the bivariate form of this model.

The mean equation:

$$R_t = \alpha + \beta' R_{t-1} + \varepsilon_t \qquad (5)$$

where:

$$R_t = \begin{bmatrix} r_{1,t} \\ r_{2,t} \\ r_{3,t} \end{bmatrix} \; ; \; \alpha = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} \; ; \; \beta = \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \\ \beta_{31} & \beta_{32} & \beta_{33} \end{bmatrix} \; ; \; \varepsilon_t = \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{bmatrix}.$$

The vector R_t represents the returns for the bond market, the foreign exchange market and the stock market, respectively, at time t. The α vector and the β matrix represent the coefficients in the mean equation and the ε_t vector represents the errors in the mean equation. In this formulation, equation (5) represents a vector autoregressive model with a single lag in the endogenous variables.

The associated variance-covariance equation is represented by:

$$\Sigma_t = C'C + A'\varepsilon_{t-1}\varepsilon_{t-1}'A + B'\Sigma_{t-1}B \qquad (6)$$

where:

$$\Sigma_t = \begin{bmatrix} \sigma_{11,t} & \sigma_{12,t} & \sigma_{13,t} \\ \sigma_{21,t} & \sigma_{22,t} & \sigma_{23,t} \\ \sigma_{31,t} & \sigma_{32,t} & \sigma_{33,t} \end{bmatrix} \; ; \; C = \begin{bmatrix} c_{11} & 0 & 0 \\ c_{21} & c_{22} & 0 \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \; ; \; A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \; ; \; B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}.$$

The variance-covariance matrix (6) yields the following equations:[17]

$$\begin{aligned}\sigma_{11,t} = c_{11}^2 + c_{21}^2 + c_{31}^2 &+ a_{11}\left(a_{11}\varepsilon_{11,t-1} + a_{21}\varepsilon_{21,t-1} + a_{31}\varepsilon_{31,t-1}\right) + \\ &a_{21}\left(a_{11}\varepsilon_{12,t-1} + a_{21}\varepsilon_{22,t-1} + a_{31}\varepsilon_{32,t-1}\right) + \\ &a_{31}\left(a_{11}\varepsilon_{13,t-1} + a_{21}\varepsilon_{23,t-1} + a_{31}\varepsilon_{33,t-1}\right) + \\ &b_{11}\left(b_{11}\sigma_{11,t-1} + b_{21}\sigma_{21,t-1} + b_{31}\sigma_{31,t-1}\right) + \\ &b_{21}\left(b_{11}\sigma_{12,t-1} + b_{21}\sigma_{22,t-1} + b_{31}\sigma_{32,t-1}\right) + \\ &b_{31}\left(b_{11}\sigma_{13,t-1} + b_{21}\sigma_{23,t-1} + b_{31}\sigma_{33,t-1}\right)\end{aligned} \tag{7}$$

$$\begin{aligned}\sigma_{22,t} = c_{22}^2 + c_{32}^2 &+ a_{12}\left(a_{12}\varepsilon_{11,t-1} + a_{22}\varepsilon_{21,t-1} + a_{32}\varepsilon_{31,t-1}\right) + \\ &a_{22}\left(a_{12}\varepsilon_{12,t-1} + a_{22}\varepsilon_{22,t-1} + a_{32}\varepsilon_{32,t-1}\right) + \\ &a_{32}\left(a_{12}\varepsilon_{13,t-1} + a_{22}\varepsilon_{23,t-1} + a_{32}\varepsilon_{33,t-1}\right) + \\ &b_{12}\left(b_{12}\sigma_{11,t-1} + b_{22}\sigma_{21,t-1} + b_{32}\sigma_{31,t-1}\right) + \\ &b_{22}\left(b_{12}\sigma_{12,t-1} + b_{22}\sigma_{22,t-1} + b_{32}\sigma_{32,t-1}\right) + \\ &b_{32}\left(b_{12}\sigma_{13,t-1} + b_{22}\sigma_{23,t-1} + b_{32}\sigma_{33,t-1}\right)\end{aligned} \tag{8}$$

$$\begin{aligned}\sigma_{33,t} = c_{33}^2 &+ a_{13}\left(a_{13}\varepsilon_{11,t-1} + a_{23}\varepsilon_{21,t-1} + a_{33}\varepsilon_{31,t-1}\right) + \\ &a_{23}\left(a_{13}\varepsilon_{12,t-1} + a_{23}\varepsilon_{22,t-1} + a_{33}\varepsilon_{32,t-1}\right) + \\ &a_{33}\left(a_{13}\varepsilon_{13,t-1} + a_{23}\varepsilon_{23,t-1} + a_{33}\varepsilon_{33,t-1}\right) + \\ &b_{13}\left(b_{13}\sigma_{11,t-1} + b_{23}\sigma_{21,t-1} + b_{33}\sigma_{31,t-1}\right) + \\ &b_{23}\left(b_{13}\sigma_{12,t-1} + b_{23}\sigma_{22,t-1} + b_{33}\sigma_{32,t-1}\right) + \\ &b_{33}\left(b_{13}\sigma_{13,t-1} + b_{23}\sigma_{23,t-1} + b_{33}\sigma_{33,t-1}\right)\end{aligned} \tag{9}$$

Equation (6) represents the BEKK formulation of the trivariate GARCH procedure,[18] which reflects the quadratic form of the multivariate GARCH, where C is a lower triangular matrix and A and B are coefficient matrices of the ARCH and GARCH components. From equations (7) to (9), the asset variances are dependent on constants,

[17] Note here that $\varepsilon_{ii} = \varepsilon_i^2$ and $\varepsilon_{ij} = \varepsilon_i \times \varepsilon_j$.

[18] For purposes of illustration, the covariance equations were omitted. See appendix for the full model consisting of three variance equations and six covariance equations.

the lag of squared residual terms, the products of lags of cross residual terms, the lags of variances and the lags of co-variances. The degree of volatility spillovers is captured by the impact of the lagged squared residuals $\varepsilon_{11,t-1}$, $\varepsilon_{22,t-1}$ and $\varepsilon_{33,t-1}$ or the effect of the lagged variances $\sigma_{11,t-1}$, $\sigma_{22,t-1}$ and $\sigma_{33,t-1}$ on the variances of the asset return at time t[19]. In comparing the two possible sets of spillover effects, Zahnd (2002) posited that the effect of the lagged variances on the current variances is delayed. He noted that shocks in asset return variances would first take effect through the squared residuals, and that the impact of the lagged variance on the current variance is reflective of a second round effect occurring within the variance equation. Specifically, a change in $\sigma_{ii,t-1}$ is partially dependent on a shock in period $t-2$, where $\varepsilon_{ii,t-1}$ is the indicator of a volatility spillover. On this basis, the impact of the lagged squared residuals provides a more accurate measure of volatility spillovers that the effect of the lagged variances.

The model coefficients specified by equations (7) to (9) reveal the impact of the squared residuals on the different asset variances. For example, a_{21}^2 represents the extent of volatility spillovers from the foreign exchange market to the bond market and a_{12}^2 reflects the extent of volatility spillovers from the bond market to the foreign exchange market. Similarly, a_{13}^2 measures the extent to which there is a volatility spillover from the bond market to the stock market and a_{31}^2 represents the extent of volatility spillovers from the stock market to the bond market. With regard to the relationship between the foreign exchange and the stock market, a_{23}^2 represents the extent to which there are volatility spillovers from the foreign exchange market to the stock market, while a_{32}^2 represents the extent to which there are volatility spillovers from the stock market to the foreign exchange market. The common market volatility effects may also be assessed. In this case, the coefficients a_{11}^2, a_{22}^2 and a_{33}^2 represent the effect of the squared bond market residual at time $t-1$ on the bond market volatility at time t, the effect of the squared foreign exchange market residual at time $t-1$ on the foreign exchange market volatility

[19] The model also allows us to measure the effect of lagged variances on the current variance.

at time t and the effect of the squared stock market residual at time $t-1$ on the stock market volatility at time t.

5. Data

The study utilises daily observations for the main Jamaica Stock Exchange (JSE) Index, the 30-day private repurchase agreement rates and the weighted average selling exchange rate to compute the continuously compounded market returns as follows:

$$r_t = 100 \times \ln\left[\frac{p_t}{p_{t-1}}\right] \tag{10}$$

In this case, r_t is the rate of return and p_t is the market price. The nominal capital pre-tax gains are computed to generate a representative asset return from each of the three major financial markets[20]. The sample period is from 07 February 2002 to 17 November 2003, representing 518 data points (see Appendix 1 for plot of data series).

The 30-day private money market rate was selected on the basis of continuity in the series and its consistency in mirroring the rates on public money and bond market securities, bank lending rates and other private rates. Using the equivalent yield transformation, the 30-day rates are converted to a daily series. At the 5 per cent level of significance the augmented Dickey Fuller rejects the null hypothesis of a unit root in the data generating process of the bond market return series. Additionally, the Jarque – Bera statistic rejects the normality hypothesis[21]. As depicted in Figure A1 (see Appendix 2), the relatively high coefficient of skewness (1.308) and kurtosis (4.584) reflect the high concentration of bond market returns in the left tail of the distribution. The low dispersion of the bond market return, relative to the other market returns is reflected in the relatively small standard deviation (0.017). The high kurtosis is typical of financial market returns and the relatively low bond market dispersion reflects the typical characteristic of fixed income markets. The Q – statistic corresponding to the series residuals reveal the existence of ARCH effects.

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[20] See data series in appendix.

[21] A normally distributed series reflects a skewness coefficient of 0 and a kurtosis of 3.

Similar to the case of the bond market return series, there is no evidence of non-stationarity at the 5.0 per cent level of significance for foreign exchange market return series. The null hypothesis of normality in the distribution is also rejected due to a significant kurtosis of 59.2 and the negative skewness in the data reflecting the impact of a small number of negative returns in the left tail of the distribution (see Figure A2 in Appendix 2). The high kurtosis is again reflecting the bunching of returns in the upper half of the distribution that is due to the depreciation bias in the domestic foreign exchange market. The foreign exchange return series also exhibits significant ARCH effects.

The Main JSE index is a market value weighted index that includes all the entities listed on the JSE and is therefore the broadest representation of prices of stocks traded on the equity market. The unit root hypothesis is also rejected for the stock market return series at the 5.0 per cent level of significance. The positive skewness and fairly high standard deviation in the data indicate the existence of a few instances of very high stock market return. The data exhibits extremely high kurtosis. Similar to the other two markets, tests of the residuals reveal significant ARCH effects.

In order to isolate the volatility transmission impact in each market due to uncertainty, it is necessary to account for the exogenous liquidity effects in the model. That is, contemporaneous and lagged volatility linkages within and between markets might be caused by significant changes in market liquidity. Thus, it is important to distinguish between these sources of volatility spillovers in order to determine the significance of the respective transmission channels. Liquidity in the model is represented by the sum of the maturities of Government of Jamaica (GOJ) bonds and Bank of Jamaica (BOJ) open market instruments, measured in Jamaica Dollars.

6. Empirical Results

Table 1 shows the common market volatility spillover estimates for the case where the model controls for liquidity[22]. The impact on all three market return volatilities arising from a shock on the previous day is statistically significant[23]. This indicates the existence of strong serial correlation in bond market returns, foreign exchange market returns and stock market returns. However, the common-market volatility spillover in the bond market is the weakest of the three markets, while the strongest occurs in the foreign exchange market.

Table 1
Volatility Transmission:
Controlling for liquidity in the model

	Bond variance	Foreign exchange variance	Stock variance
Bond variance	0.0022562866	0.0003690961	0.0024754751
Foreign exchange variance	0.0000000438	0.0026676487	0.0000154207
Stock variance	0.0000000016	0.0000000936	0.0025812333

The cross-market volatility spillover effects are weaker than for the common market. The strongest spillover occurs from the bond market to the stock market. The volatility spillover effects between these two markets are asymmetric, with the smallest spillover arising from the stock market to the bond market. The relatively weak volatility spillover from the stock market to the bond market is consistent with the risk differential. As illustrated in Appendix 1, stock market returns are more volatile than bond market returns. This is partially due to the fact that the Jamaican bond market is far larger than the domestic stock market[24].

[22] GARCH estimation results are presented in Tables 3.1 and 3.2 in the Appendix. The statistically significant coefficients demonstrate the existence of cross-market and common market volatility spillovers. This is consistent with a priori expectations. In the case where the model controls for liquidity, bond maturities are used as an explanatory variable in the mean equation of the model. In this case, the variable absorbs the liquidity impact that would, otherwise, be reflected in the variance equation.

[23] Engle and Sheppard (2001) note the difficulties in interpreting the absoluteness of the volatility spillover coefficients derived from the BEKK model. However, the relative magnitude of the spillover coefficients is useful in comparing common market and cross-market spillover effects.

[24] Bank of Jamaica's Quarterly Monetary Policy Report (March 2004).

The second largest volatility spillover effect is observed in the impact of the bond market on the foreign exchange market returns. Similar to the case of the bond and the stock markets, there is a marked asymmetry between the bond market and foreign exchange market spillovers[25]. The relatively strong volatility transition from the bond market returns to foreign exchange market returns suggests that an information event that alters the expectation about returns in the bond market will significantly influence demand and trading in the foreign exchange market.

Additionally, there exists strong volatility spillovers from the foreign exchange market to the stock market. This spillover is the third largest among the cross-market volatility spillover effects. In contrast, the volatility spillover from the stock market to the foreign exchange market is the third lowest. This may be explained by the fact that the stock market returns are more volatile than the foreign exchange market returns, as evidenced in Appendix 1.

6.1 Impulse response analysis

A more complete understanding of volatility spillovers can be attained from an impulse response assessment by applying a simple three variable vector autoregressive model[26]. Figure 1 depicts the response of bond market volatility to one-day shocks to the three major markets. It reflects the relatively significant one day lagged effect of bond market volatility on itself, which dies out within two days of the initial shock. The response of bond market volatility to volatility in the foreign exchange market is most intense on the second day and dies out within six to seven days. The figure also reveals that stock market volatility exerts the least influence on bond market volatility over a ten-day period. This response peaks at around the third day and dies out after four to five days.

[25] The coefficient that reflects the extent of volatility spillovers from the foreign exchange market to the bond market is the second lowest volatility spillover coefficient (see Table 1).

[26] In this application the impulses are standardized as a percentage of the initial common market effect of the market under consideration. Due to the mechanics of the model, it is highly unlikely that an initial common market impulse may evolve into a cross-market impulse in a future period. The reverse is also true.

Figure 1
Response of bond market variance to lagged market variances accounting for liquidity in the model

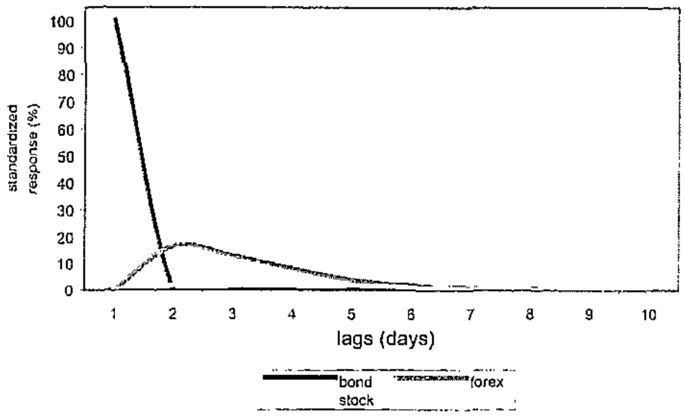


Figure 2 shows the response of foreign exchange market volatility to shocks originating in the three major markets. The strong expectations component of foreign exchange market returns is reflected in the relatively significant five to six day impact of lagged foreign exchange return volatility on current volatility. The bond market volatility impact on foreign exchange market returns volatility dies out in two days. The stock market return volatility, however, has a lower but more sustained impact on return volatility in the foreign exchange market. This impact dies out in approximately five to six days.

Figure 2
Rsponse of forex market variance to lagged market variances accounting for liquidity in the model

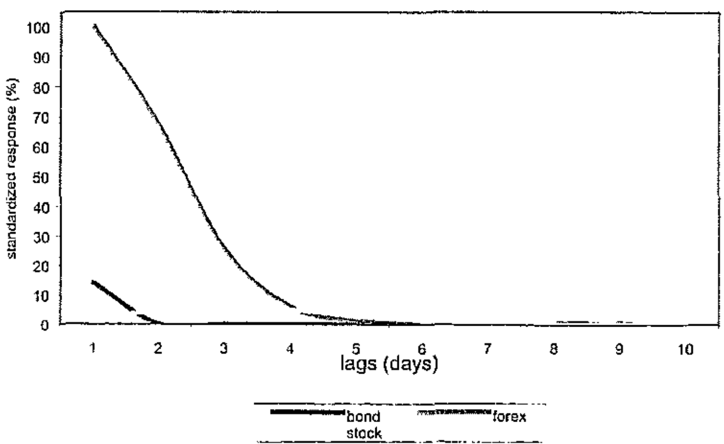
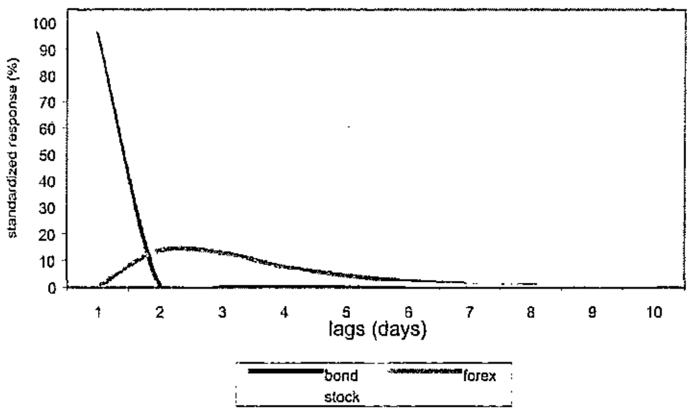


Figure 3 shows the extent to which the lagged stock market variance influences its current variance. This response falls off steeply over the first two days, but persist at a relatively low level for another five to six days. The diagram reflects the relatively brief response of stock market return volatility emanating from the bond market, dying out within two days of the initial shock. Although beginning at a relatively low level, the response of the stock market return volatility to a shock in the foreign exchange market peaks approximately three days after the initial impulse and eventually dies out after six to seven days.

Figure 3
Response of stock market variance to lagged market variances accounting for liquidity in the model



The above results suggest that volatility spillovers emanating from the bond market usually die out within two days. The brevity of this impulse indicates the efficiency of these markets in processing new information originating in the bond market.

6.2 Liquidity impact

To measure the impact of a change in liquidity conditions, the model is re-run without controlling for the maturity profile of Government and Central Bank fixed income securities. That is, the results reflect the full impact of spikes in liquidity emanating from blocks of securities. Table 2 reveals that not controlling for liquidity effects results in a 0.02 per cent decline in the bond market volatility spillover and an increase of 0.01 per cent in the common market volatility spillover for both the foreign exchange and stock markets. However, all the common market liquidity effects were negligible.

Table 2
Volatility Transmission:
Not controlling for liquidity in the model

	Bond variance	Foreign exchange variance	Stock variance
Bond variance	0.0022557325	0.0003708769	0.0024911296
Foreign exchange variance	0.0000000439	0.0026678871	0.0000155272
Stock variance	0.0000000016	0.0000000944	0.0025815648

The difference in the results between Table 1 and Table 2 also indicates the low liquidity impact on cross-market volatility spillovers[27]. The omission of liquidity effects from the model had the smallest impact on the volatility spillovers to the bond market. Maturities resulted in a 0.23 per cent increase in the spillover from the foreign exchange market to the bond market and no increase in transmission from the stock market to the bond market. There is a similar volatility impact of foreign exchange and bond market spillovers to the stock market. Not controlling for maturities leads to a 0.63 per cent increase in the extent to which information events concerning the bond market spill over to the volatility in the stock market, and a 0.69 per cent increase in the spillover from the foreign exchange market to the stock market. The liquidity impact on volatility spillovers to the foreign exchange market was varied. The exclusion of liquidity effects results in a 0.85 per cent increase in the spillover from the stock market to the foreign exchange market and a 0.48 per cent increase in spillover from the bond market to the foreign exchange market.

[27] When there is no control for liquidity the ranks are preserved.

6.2.1 Liquidity impact and impulse response

Figure 4
Response of bond market variance to lagged market variances not accounting for liquidity in the model

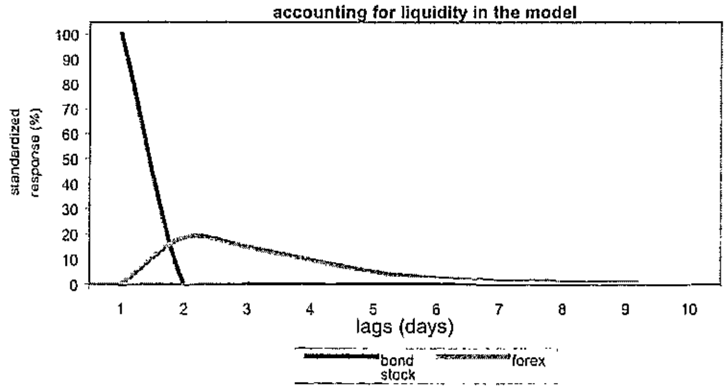


Figure 5
Response of forex market variance to lagged market variances not accounting for liquidity in the model

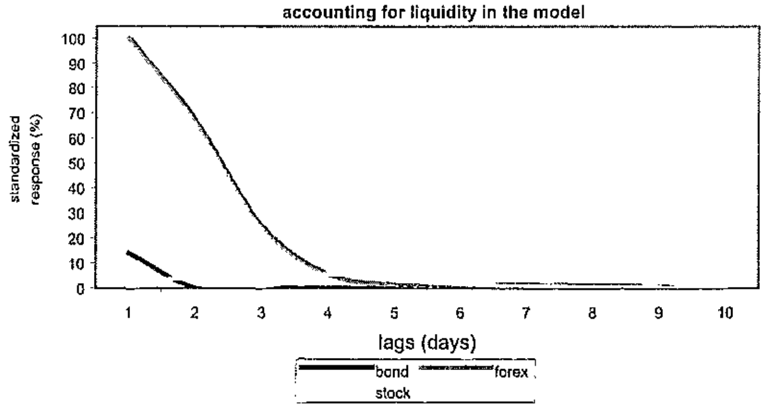
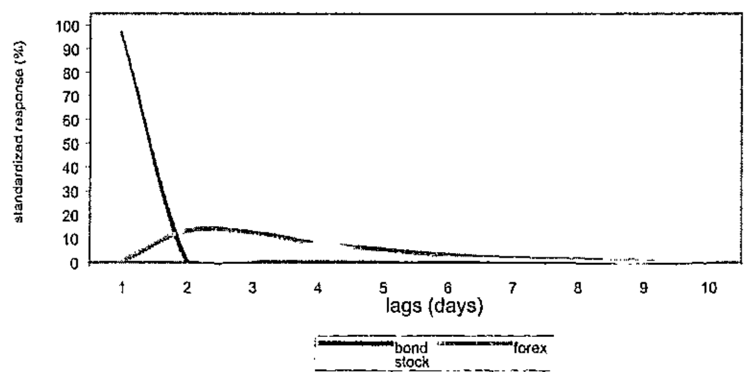


Figure 6
Response of stock market variance to lagged market variances not accounting for liquidity in the model



Figures 4, 5 and 6 show that there are considerable similarities between the impulses in the absence of the liquidity impact and with liquidity impact. These similarities imply that, although most of the initial volatility spillovers reflect slight differences with the consideration of liquidity in the financial system, the responses on the market volatilities considered over a ten-day range have a negligible impact. This may be indicative of the extent to which monetary authorities employ effective policies to limit the occurrence of second round spillovers and hence greater impulses over higher lags.

7. Summary and policy recommendation

In summary, the results of the model indicate that there are generally high levels of common market volatility spillover relative to cross-market spillovers within the Jamaican financial system. Of the three markets, the foreign exchange market exhibits the most pronounced common market volatility spillovers, followed by the stock market. The strong common market spillover in these two markets, relative to that of the bond market reflects the uncertainty and herd behaviour that often characterise these risky markets.

The strongest cross-market effect occurs from the bond market to the stock market. This result is in sharp contrast to the weak volatility spillover from the stock market to the bond market. The second highest source of cross-market volatility propagation occurs from the bond market to the foreign exchange market. The relatively low spillover from foreign exchange market to the bond market also suggests some degree of asymmetry. The third highest volatility spillover channel is represented by volatility propagation from the foreign exchange market to the stock market. This channel reflects the lowest case of asymmetry.

Considering the different effects of spillover events, impulse response analyses reveal that the volatility spillovers from the bond market usually die out within two days, those from the foreign exchange market die out within five to seven days and spillovers emanating from the stock market usually take four to six days to die out. Similar to the case of common market volatility spillovers, this result is consistent with the characteristic stability of the bond market, relative to the foreign exchange and stock markets.

When bond maturities are allowed to influence the market return volatilities[28], there are only minor differences in the various volatility spillover channels. Generally, the model indicates that when liquidity is allowed to influence return volatilities, there is a decline in the common market volatility spillover exhibited in the bond market. On the other hand, the liquidity causes an increase in foreign exchange market and stock market return volatilities. In terms of the cross-market spillover effects, changes in the liquidity conditions have a lesser impact on spillovers to the bond market than for the foreign exchange and the stock markets. Generally, the changes in liquidity do not have a significant impact on the duration of the volatility spillovers. This result suggests that monetary policy is successful in restricting the impact of volatility impulses within and between markets that stem from liquidity effects.

The results from this study highlight that each of the three major Jamaican financial markets is characterised by quantifiable uncertainty linkages with each other. Empirical results indicate that financial system participants, including the regulators, must consider the intricate market connections. These involve uncertainty spillovers that are relevant to financial system stability and monetary transmission.

With regard to the stability of the financial system, market risk is of particular importance. The market risk reflects the volatility, as well as the co-volatility among the different markets. Increased volatility in a single market, by itself, may not pose a serious threat to the investor's portfolio, as effective diversification strategies may help to minimize the increase in risk exposure. A more significant threat to systemic financial stability may be the existence of volatility linkages that may cause difficulties in diversifying portfolio risks[29]. These linkages increase the probability of systemic instability and therefore must be monitored by financial system regulators. The results from the study suggest that, based on the volatility linkages between the major financial

[28] In this case, the model does not control for liquidity.

[29] Negative asset return correlation may be associated with positive volatility spillovers.

markets, risk-based capital requirements for financial entities should be computed with consideration of the correlations between the different sources of market risk. The recent amendment to the BOJ Act that accounts for foreign exchange risk in the computation of capital requirements does not consider the linkage between the equity, foreign exchange and bond markets.[30] A more dynamic approach in accounting for market risk would involve simultaneous consideration for other types of market risks as suggested by the Basle capital requirements.

The results reveal that the foreign exchange market exhibits the strongest and most sustained common market volatility spillover effects. Given the importance of foreign exchange market stability in attaining overall price stability in the Jamaican economy, the results support continuous emphasis on the timely moderating of foreign exchange market uncertainties. Interestingly, the model also reveals that even though the initial spillovers from the bond market are usually significant, these spillovers usually die out in a short time. This result implies that the pursuit of foreign exchange market stability could take place at the expense of short-term disruptions in the bond market.

[30] This law was passed in October 2003.

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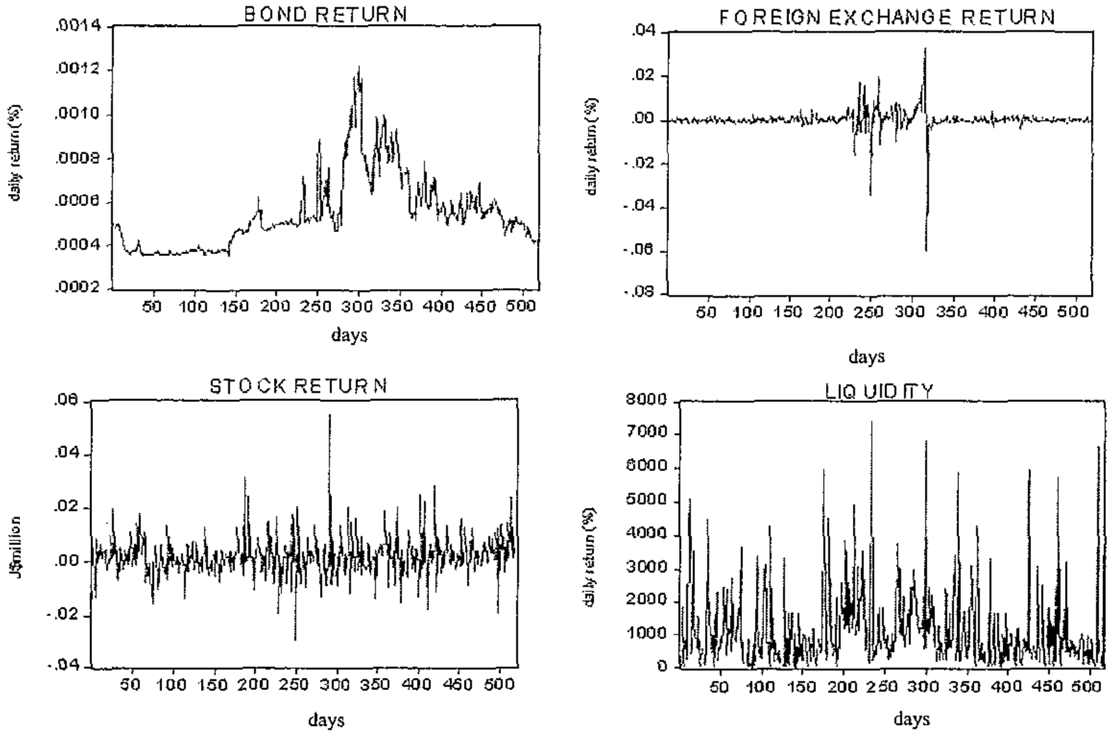
Appendix 1

Plot of daily return series and bond market liquidity



Appendix 2
Histograms and descriptive statistics

Figure 2.1 - Daily Bond Market Return
February 2002 to March 2004

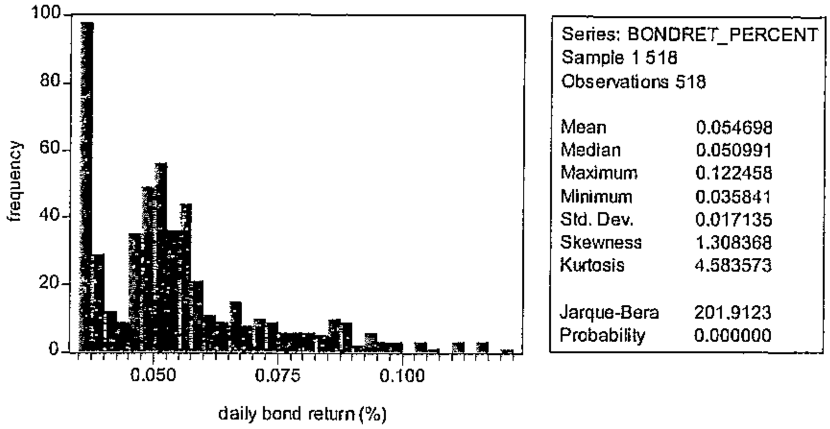


Figure 2.2 - Daily Foreign Exchange Market Return
February 2002 to March 2004

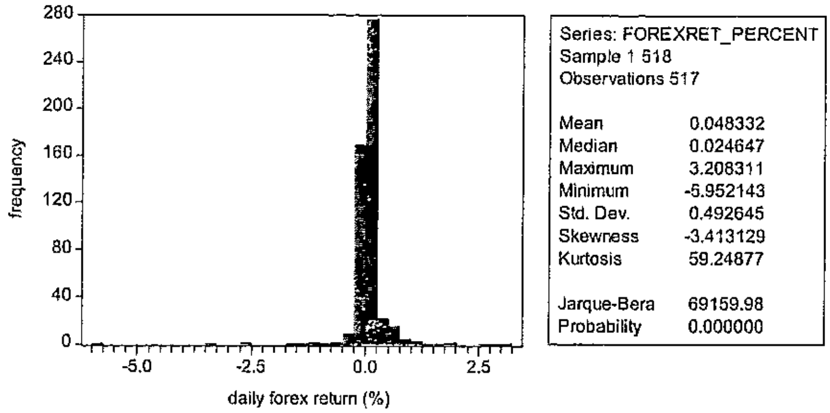
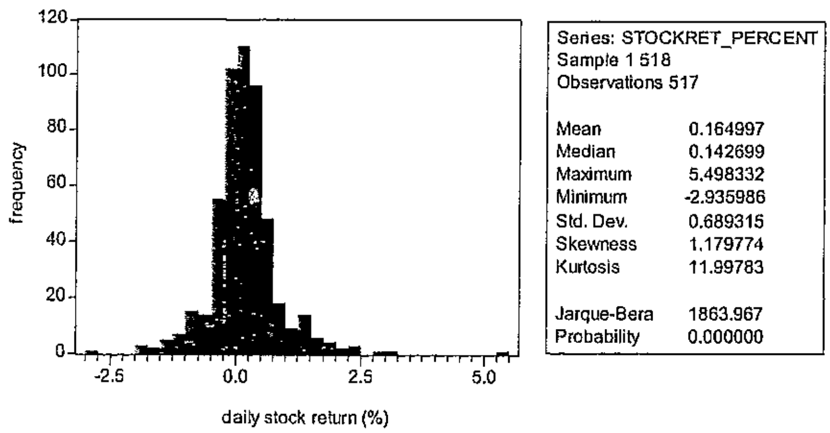


Figure 2.3 - Daily Stock Market Return
February 2002 to March 2004



Appendix 3

Model results

Table 3.1
GARCH results controlling for liquidity effects

Coefficients	Value	P-value
	A-Matrix	
a11	0.047500	0.0000
a21	0.000209	0.0000
a31	-0.000040	0.0000
a12	-0.019212	0.0000
a22	0.051649	0.0000
a32	-0.000306	0.0000
a13	0.049754	0.0000
a23	-0.003927	0.0000
a33	0.050806	0.0000
	B-Matrix	
b11	0.056088	0.1761
b21	0.005271	0.0000
b31	-0.006141	0.0437
b12	6.099788	0.0000
b22	0.093083	0.0000
b32	-0.103554	0.0000
b13	-6.597950	0.0000
b23	-0.106508	0.0000
b33	0.190792	0.0001

Table 3.2
GARCH results not controlling for liquidity effects

Coefficients	Value	P-value
	A-Matrix	
a11	0.047495	0.0000
a21	0.000210	0.0000
a31	-0.000040	0.0000
a12	-0.019258	0.0000
a22	0.051652	0.0000
a32	-0.000307	0.0000
a13	0.049911	0.0000
a23	-0.003940	0.0000
a33	0.050809	0.0000
	B-Matrix	
b11	0.057167	0.1512
b21	0.005297	0.0000
b31	-0.006351	0.0598
b12	6.106732	0.0000
b22	0.093223	0.0000
b32	-0.103882	0.0000
b13	-6.571240	0.0000
b23	-0.106705	0.0000
b33	0.191231	0.0006

Appendix 4

Covariances derived from the BEKK representation of the Multivariate GARCH (1,1)

$$
\begin{aligned}
\sigma_{12,t} = c_{21}c_{22} + c_{31}c_{32} + {} & a_{12}\left(a_{11}\varepsilon_{11,t-1} + a_{21}\varepsilon_{21,t-1} + a_{31}\varepsilon_{31,t-1}\right) + \\
& a_{22}\left(a_{11}\varepsilon_{12,t-1} + a_{21}\varepsilon_{22,t-1} + a_{31}\varepsilon_{32,t-1}\right) + \\
& a_{32}\left(a_{11}\varepsilon_{13,t-1} + a_{21}\varepsilon_{23,t-1} + a_{31}\varepsilon_{33,t-1}\right) + \\
& b_{12}\left(b_{11}\sigma_{11,t-1} + b_{21}\sigma_{21,t-1} + b_{31}\sigma_{31,t-1}\right) + \\
& b_{22}\left(b_{11}\sigma_{12,t-1} + b_{21}\sigma_{22,t-1} + b_{31}\sigma_{32,t-1}\right) + \\
& b_{32}\left(b_{11}\sigma_{13,t-1} + b_{21}\sigma_{23,t-1} + b_{31}\sigma_{33,t-1}\right)
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{13,t} = c_{31}c_{33} + {} & a_{13}\left(a_{11}\varepsilon_{11,t-1} + a_{21}\varepsilon_{21,t-1} + a_{31}\varepsilon_{31,t-1}\right) + \\
& a_{23}\left(a_{11}\varepsilon_{12,t-1} + a_{21}\varepsilon_{22,t-1} + a_{31}\varepsilon_{32,t-1}\right) + \\
& a_{33}\left(a_{11}\varepsilon_{13,t-1} + a_{21}\varepsilon_{23,t-1} + a_{31}\varepsilon_{33,t-1}\right) + \\
& b_{13}\left(b_{11}\sigma_{11,t-1} + b_{21}\sigma_{21,t-1} + b_{31}\sigma_{31,t-1}\right) + \\
& b_{23}\left(b_{11}\sigma_{12,t-1} + b_{21}\sigma_{22,t-1} + b_{31}\sigma_{32,t-1}\right) + \\
& b_{33}\left(b_{11}\sigma_{13,t-1} + b_{21}\sigma_{23,t-1} + b_{31}\sigma_{33,t-1}\right)
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{21,t} = c_{22}c_{21} + c_{32}c_{31} + {} & a_{11}\left(a_{12}\varepsilon_{11,t-1} + a_{22}\varepsilon_{21,t-1} + a_{32}\varepsilon_{31,t-1}\right) + \\
& a_{21}\left(a_{12}\varepsilon_{12,t-1} + a_{22}\varepsilon_{23,t-1} + a_{32}\varepsilon_{32,t-1}\right) + \\
& a_{31}\left(a_{12}\varepsilon_{13,t-1} + a_{22}\varepsilon_{23,t-1} + a_{32}\varepsilon_{33,t-1}\right) + \\
& b_{11}\left(b_{12}\sigma_{11,t-1} + b_{22}\sigma_{21,t-1} + b_{32}\sigma_{31,t-1}\right) + \\
& b_{21}\left(b_{12}\sigma_{12,t-1} + b_{22}\sigma_{22,t-1} + b_{32}\sigma_{32,t-1}\right) + \\
& b_{31}\left(b_{12}\sigma_{13,t-1} + b_{22}\sigma_{23,t-1} + b_{32}\sigma_{33,t-1}\right)
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{23,t} = c_{32}c_{33} + {} & a_{13}\left(a_{12}\varepsilon_{11,t-1} + a_{22}\varepsilon_{21,t-1} + a_{32}\varepsilon_{31,t-1}\right) + \\
& a_{23}\left(a_{12}\varepsilon_{12,t-1} + a_{22}\varepsilon_{22,t-1} + a_{32}\varepsilon_{32,t-1}\right) + \\
& a_{33}\left(a_{12}\varepsilon_{13,t-1} + a_{22}\varepsilon_{23,t-1} + a_{32}\varepsilon_{33,t-1}\right) + \\
& b_{13}\left(b_{12}\sigma_{11,t-1} + b_{22}\sigma_{21,t-1} + b_{32}\sigma_{31,t-1}\right) + \\
& b_{23}\left(b_{12}\sigma_{12,t-1} + b_{22}\sigma_{22,t-1} + b_{32}\sigma_{32,t-1}\right) + \\
& b_{33}\left(b_{12}\sigma_{13,t-1} + b_{22}\sigma_{23,t-1} + b_{32}\sigma_{33,t-1}\right)
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{31,t} = c_{33}c_{31} + {} & a_{11}\left(a_{13}\varepsilon_{11,t-1} + a_{23}\varepsilon_{21,t-1} + a_{33}\varepsilon_{31,t-1}\right) + \\
& a_{21}\left(a_{13}\varepsilon_{12,t-1} + a_{23}\varepsilon_{22,t-1} + a_{33}\varepsilon_{32,t-1}\right) + \\
& a_{31}\left(a_{13}\varepsilon_{13,t-1} + a_{23}\varepsilon_{23,t-1} + a_{33}\varepsilon_{33,t-1}\right) + \\
& b_{11}\left(b_{13}\sigma_{11,t-1} + b_{23}\sigma_{21,t-1} + b_{33}\sigma_{31,t-1}\right) + \\
& b_{21}\left(b_{13}\sigma_{12,t-1} + b_{23}\sigma_{22,t-1} + b_{33}\sigma_{32,t-1}\right) + \\
& b_{31}\left(b_{13}\sigma_{13,t-1} + b_{23}\sigma_{23,t-1} + b_{33}\sigma_{33,t-1}\right)
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{33,t} = c_{33}c_{32} + {} & a_{12}\left(a_{13}\varepsilon_{11,t-1} + a_{23}\varepsilon_{21,t-1} + a_{33}\varepsilon_{31,t-1}\right) + \\
& a_{22}\left(a_{13}\varepsilon_{12,t-1} + a_{23}\varepsilon_{22,t-1} + a_{33}\varepsilon_{32,t-1}\right) + \\
& a_{32}\left(a_{13}\varepsilon_{13,t-1} + a_{23}\varepsilon_{23,t-1} + a_{33}\varepsilon_{33,t-1}\right) + \\
& b_{12}\left(b_{13}\sigma_{11,t-1} + b_{23}\sigma_{21,t-1} + b_{33}\sigma_{31,t-1}\right) + \\
& b_{22}\left(b_{13}\sigma_{12,t-1} + b_{23}\sigma_{22,t-1} + b_{33}\sigma_{32,t-1}\right) + \\
& b_{32}\left(b_{13}\sigma_{13,t-1} + b_{23}\sigma_{23,t-1} + b_{33}\sigma_{33,t-1}\right)
\end{aligned}
$$