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FLOATING EXCHANGE RATES AND EXCHANGE RATE VOLATILITY IN THE CARIBBEAN

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Volatility Clusters and Chaos under Floating Exchange Rate Regimes in the Caribbean: Evidence from Jamaica, Guyana and Trinidad and Tobago

by

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Introduction

Within the last five years, three Caribbean countries have adopted floating exchange rate systems as the philosophy of trade and financial liberalisation has gained widespread acceptance. Some proponents argue that there is a strong likelihood that the floating rate regime adopted will lead to greater volatility and increased instability in exchange rate behaviour in the Caribbean environment and that this is likely to have an adverse effect on trade and investment in the short and long term. Others advocate that the floating exchange rate regime is likely to increase the efficiency and competitiveness of the foreign exchange market, obviating the need to develop complex, bureaucratic and irresponsive exchange rate management strategies. The floatation of the currencies of Jamaica, Guyana and Trinidad and Tobago in the decade of the early 1990s has left two burning questions in the minds of the monetary authorities. First, can small changes in the exchange rate lead to wild and explosive movements (usually depreciations) in the value of the currency in ensuing periods? Second, to what extent is prediction of the movement of the exchange rate possible in the new floating dispensation?

This paper attempts to provide some answers to these burning issues. Section 1 presents a brief historical review of the evolution of exchange rate policy in Jamaica, Guyana and Trinidad and Tobago. Section 2 discusses the concept of volatility and presents empirical evidence of the distribution of daily buying and selling rates in the three CARICOM member states. Volatility clusters are explored through the use of the Generalised Autoregressive Conditional Heteroscedastic [GARCH] models allowing for thick tails via the Student-t distribution. Section 3 explores whether or not the fluctuations in the exchange rates are predictable through the use of the concept of chaos. In particular, the concepts of fractal dimension (correlation dimension) and lyapunov exponents are explored and the BDS statistic is employed to test for the existence of low-dimensional chaotic behaviour. The final section of the paper presents some concluding remarks.

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Section 1: Evolution of Exchange Rate Regimes in the Caribbean

In the decade of the 1950s exchange rate arrangements in the Caribbean were supervised by a British Caribbean Currency Board. Trinidad and Tobago, Jamaica and Guyana formed part of the sterling Area. The advent of the independence movement in the early and mid-1960s witnessed the establishment of individual central banks with the power to issue their own monies. The individual currencies were initially pegged to the pound sterling but the devaluation of sterling in November 1967 led to major questions in the Caribbean about the appropriateness of the sterling peg for the Caribbean. By 1972, the pound came under such intense pressure, prompting the British authorities to allow it to float. By then of course all the major developed countries had shifted from fixed regimes to a floating mechanism.

The issue of the choice of appropriate exchange rate mechanism to facilitate development was occupying the attention of the various central banks in the Caribbean environment. Indeed, there was considerable debate about the merits of fixed *vis-à-vis* floating exchange rates throughout the developing world. The major argument against adopting a floating exchange rate which surfaced at the time revolved around the question of risk and uncertainty in international transactions. The view was held that for small developing countries which depended heavily on trade, exchange rate uncertainty made it more difficult for exporters and importers to enter into long-term commitments. Recognizing at the same time the growing importance of their trading relations with the United States, Caribbean countries opted to peg to the United States dollar. In January 1975, the Jamaican dollar was fixed at US\$1: J\$1.10. Guyana followed suit in October 1975 and pegged at US\$1: G\$2.55 while Trinidad and Tobago pegged its dollar in May 1976 at US\$1: TT\$2.40.

By the late 1970s however, the decline in the prices of bauxite and sugar led to severe balance of payments problems in Guyana and Jamaica. Indeed, over the period 1981-1989 Jamaica and Guyana incurred cumulative balance of trade deficits of US\$4,660 million and

US\$181.0 million, respectively. The decade of the 1980s was characterised by heavy reliance on external borrowing, large budget deficits and rising inflation. This period also marked the beginning of a series of exchange rate devaluations as countries attempted to increase foreign exchange earnings, reduce the balance of payments deficits and restore stability in the economic system. Table 1 presents a short chronology of exchange rate realignments under a fixed regime.

The first half of the 1990s has witnessed the onset of trade and financial liberalisation on a global scale. The formation of the World Trade Organisation has completed the global institutional trilogy which has established the new rates for global trade and finance. The widespread acceptance of economic liberalisation in the Caribbean has also pervaded the foreign exchange market. Given the problems experienced in managing their fixed rate regimes, Jamaica, Guyana and Trinidad and Tobago were gently coerced into adopting flexible exchange rate systems. 1 This step towards liberalisation is expected to ease the chronic balance of payments problem, obviate the need for reserves, support an export oriented strategy and contribute to greater economic stability. This period of floatation has however been somewhat characterised by large depreciations in the value of the exchange rate and has led to even greater speculation about the efficiency of floating regimes. The spectre of uncertainty in the rate has fuelled volatility and has led to even greater difficulty in the ability of the monetary authorities to make both long and short term predictions about the magnitude and direction in the rate of change in the various buying and selling rates. The million dollar question which still remains unanswered is whether these rates can be reasonably predicted with the existing theoretical knowledge and modelling tools?

¹ In February 1991, Guyana liberalised its foreign exchange market allowing its currency to float freely. By September of the same year, the Jamaican foreign exchange market was liberalised while in April 1993, some nineteen months later, the Trinidad and Tobago dollar was allowed to float against the major reserve currencies.

Section 2: Exchange Rate Behaviour in the Caribbean: Empirical Evidence

For the purpose of this analysis the exchange rate is defined as the number of units of domestic currency that are required to purchase one unit of foreign currency. Daily exchange buying and selling rates, spanning the period April 13 - October 18, 1996, were used in the empirical calculations. Several difficulties were experienced initially in compiling a consistent set of rates across all three countries since holiday periods did not necessarily coincide during each given year. In computing the empirical moments, it was far more convenient to utilise foreign exchange returns rather than the level rate. These returns were measured by the logarithmic difference of the level rate:-

$$Log_{e}(S_{t}) - Log_{e}(S_{t-1}) = Log\left(\frac{S_{t}}{S_{t-1}}\right)$$

where S_t represents either the nominal buying or selling rate of each individual currency $vis-\dot{a}-vis$ a reserve currency. In actuality, this formula gives the continuous rate of change of each of the respective currencies under study $vis-\dot{a}-vis$ the United States dollar.

In order to glean a clearer picture of the behaviour of the exchange rates it is necessary to study the empirical moments of the various rates of change. These moments were computed for the entire sample period (April 13 -October 18, 1996) and as well for the individual years 1993, 1994, 1995 and 1996. These individual years allows us to examine how the distribution of the rate of changes varies as the period of liberalisation deepens. Table 2 presents the empirical moments for the individual years while Table 3 provides data for the whole period.

Several interesting findings can be gleaned from an analysis of Table 2. For the year 1993, the coefficients of kurtosis which measure the peakedness of the data are relatively large in all cases although the coefficient of kurtosis for the selling rate of the US\$: GY\$ is much larger than those for the other currencies. The coefficients of skewness were relatively

small in most cases except for the selling rate of the US\$: TT\$ which was positively skewed. Given the results, the normal distribution may only be a useful candidate for RLGYUSB and RLJJUSB. Larger coefficients of kurtosis were recorded for the remaining years 1994, 1995 and 1996, respectively, indicating high levels of leptokurtosis in these periods. It is tempting to assume here based on the results that the various exchange rates variables exhibit more probability mass in the tails although one may not be too far-fetched. This would signify that the distributions are 'fat-tailed' and imply that extremely high and low realizations occur more frequently than under the hypothesis of normality. However, De Vries (1994) has cautioned that an important distinction must be made between 'fat-tailed' and 'thin-tailed' distributions. In the former case, the tails decline exponentially while in the latter they decline by some given power. There is still some controversy on the precise means by which these tails should be estimated and some useful approaches have surfaced in the writings of Smith (1987) and Haan (1990).

A cursory examination of the plots of the logarithmic differences of the varioùs buying and selling rates indicate periods of unusually large volatility followed by periods of relative tranquillity. These periods of turbulence and quiescence represent volatility clusters and are often associated with the 'fat-tail' phenomenon. The recent econometric literature has demonstrated that this type of volatility clustering may be modelled by conditional distributions which allow for heteroscedasticity in the variance. This issue is well discussed in the works of Engle (1982), Bollerslev (1986), Bollerslev, Chou and Kroner (1992) and Bera and Higgins (1995).

2.1 The GARCH Methodology

The GARCH method as proposed by Bollerslev (1986) is represented as follows:-

$$\begin{aligned} y_t \middle| \Phi_{t-1} &\sim (V_t, h_t) \\ V_t &= g(X; \theta) + u_t \\ u_t &= \varepsilon_t - \sum_{j=1}^q \gamma_j \varepsilon_{t-j} \\ \varepsilon_t \middle| \Phi_{t-1} &\sim \Omega(0, h_t) \\ h_t &= f(\varepsilon_{t-1}, h_{t-1}) \end{aligned}$$

It is often assumed that the mean process is linear and that the white noise innovations follow a normal distribution. If the conditional variance is written in the following form:-

$$h_{t} = \alpha_{0} + \sum_{j=1}^{q} \alpha_{j} \varepsilon_{t-j}^{2} + \sum_{j=1}^{p} \beta_{j} h_{t-q}$$
where $p \ge 0$, $q > 0$

$$\alpha_{0} > 0$$
, $d_{j} \ge 0 \ (j = 1, ..., q)$

$$\beta_{j} \ge 0 \ (j = 1, ..., p)$$

the model is a GARCH (p,q) process.² The key feature of this model is the fact that the conditional variance follows an ARMA(q,p) process. It is also useful to note that the model is only well-defined if the coefficients of the infinite autoregressive component are non-negative and as well if the roots of the moving average polynomial of squared innovations lie outside the unit circle. Given the fact that most of the data sets exhibited high levels of kurtosis, a decision was made to utilise a GARCH (1,1) with the error following a Student-t model:-.

$$Z_{t} = 100\log\left(\frac{S_{t}}{S_{t-1}}\right)$$

$$Z_{t} = \beta_{0} + \beta_{1}Z_{t-1} + \beta_{2}Z_{t-2} + \dots + \beta_{p}Z_{t-p}$$

$$\varepsilon_{t} | \Phi_{t-1} \sim t(0, h_{t}, \delta)$$

$$h_{t} = \alpha_{0} + \alpha_{1}\varepsilon_{t-1}^{2} + \beta_{1}h_{t-1}$$

² When p=0 the process reduces to an ARCH(q) process while for p=q=0 the process becomes white noise.

where δ is our degrees of freedom. The log-likelihood of the Student-t model is given by the following expression (see Bollerslev (1987)):-

$$\log L = \sum_{t=1}^{n} \left\{ \log \Gamma\left(\frac{\nu+1}{2}\right) - \log \Gamma\left(\frac{\nu}{2}\right) - \frac{1}{2}\log(\nu-2)h_{t} \right\}$$

$$\left\{ -\frac{1}{2}(\nu+1)\log\left[1 + \varepsilon_{t}^{2}h_{t}^{-1}(\nu-2)^{-1}\right] \right\}$$

Of course, one difficulty which arises empirically revolves around the question of the determination of the lag lengths for Z_t , the autoregressive process of order, p. Such a problem can be solved by examination of the structure of the ACF and PACF functions as proposed by Box and Jenkins (1976) or by employing either one of the Schwarz, Akaike or Hannan-Quinn criterion.

Section 3: Are Exchange Rates in the Caribbean chaotic?

Most of the economic analysis of exchange rates have been performed on systems which are linear. Whereas the assumption of linearity has been quite convenient both analytically and empirically, it has generally failed to account for sudden regime shifts which lead to wild perturbations in systems. The theory of linear systems assumes that there are three states. Systems can be stable (when perturbed the system settles down to some stable value), oscillate (when perturbed the system settles into a periodic cycle) or unstable (when perturbed the movements of the system increase or decrease continually by large magnitudes). The basic presumption which underlies all linear systems is the philosophy that all phenomenon belonging to nature are guided by simple laws. Naturally, the basic advantage of linear systems is the ease with which behaviour of the system can be predicted from one time period to the next.

The recent global changes brought about by trade and financial liberalisation and the phenomenon of globalization are making forecasts of the exchange rate increasingly more difficult. In fact, the decade of the 1990s is characterised so far by abrupt and violent changes in several economic magnitudes, including, of course, the exchange rate. The traditional laws of nature which are based on linear paradigms are finding it increasingly more difficult to predict or explain with any degree of accuracy the unfolding economic landscape. Among traditionalists in the field of econometrics, there is still a strong belief that the principles developed from Newtonian physics and Euclidean geometry are valid and that the chaotic behaviour which is observed is an aberration from this reality. Chaos³ to traditionalists is therefore a state in which the natural (linear) laws of evolution are not obeyed.

Within recent times however, considerable attention has been devoted to the mathematics of dynamical systems in order to explain these changes. This mathematics emphasizes the geometry of fractal processes and seeks to explain the phenomenon of chaos. Although there is no unique definition of chaotic behaviour it is now accepted that such behaviour is characterised by:

- no bounded steady state
- no single equilibrium point
- no periodic or quasi-periodic points

Moreover, the spectrum associated with such systems tends to have a broad band noise-like component. Peters (1991) describes chaotic systems as those which have a fractal dimension and which display sensitive dependence to initial conditions. In a dynamical system the concept of a dimension defines the number of state variables that are used to describe the dynamics of the system. It is often used to quantify the complexity of an

³ The term was initially coined by Li and Yorke (1975) although its origins can be traced to the works of Poincare (1952) and Lorenz (1963).

attractor. A fractal⁴ dimension is one which is defined by non-integer values. One of the key aspects of a fractal attractor⁵ is that equilibrium is associated with a region in phase space rather than with a single point (point attractor) or limit cycle as is typically assumed in economic analysis. Indeed, if the behaviour of a system is non-periodic, then point attractors and limit cycles are incapable of defining the relevant dynamics.

If one is to understand therefore the dynamics of exchange rate movements then there is obviously a need to determine whether the system is driven in any way by an underlying chaotic attractor. Moreover, an important question which arises is the extent to which the time series are generated by non-linear deterministic laws of motion. It is important to realise therefore from the outset that in reality one may not always be observing a stochastic process. Rather, a deterministic mechanism may be at work generating behaviour which looks random. Bera and Higgins (1995) allude to this important point when they argue that the standard statistical tests used in time series and econometric analysis often fail to reject random behaviour.

3.1 Measures of Chaotic Behaviour

In order to measure the existence of chaos in a system one needs to determine whether the system has a fractal dimension and whether or not it displays sensitive dependence to initial conditions.

⁴ A fractal is in reality a number which quantitatively describes how an object fills its space. In euclidean geometry, objects are solid and continuous and have no holes or gaps (i.e. they have integer dimension).

⁵ An attractor is simply the level that a system reverts to after the effects of perturbation of the system dies away.

3.1.1 Fractal Dimension

One of the most useful practical methods for determining the fractal dimension⁶ of a system was developed by Grassberger and Procaccia (1983). These authors approximate the fractal dimension by use of the correlation dimension. This dimension measures how densely the attractor fills its phase space by determining the probability that a given point will lie a certain distance from another.

Consider an observed series, $\{x_i:1,\dots,T\}$. The correlation integral, $C_N(\varepsilon,T)$, can be written as follows:-⁷

$$C_{N}(\varepsilon,T) = \frac{2}{T_{N}(T_{n}-1)} \sum_{i\leq s} I_{s}(x_{i}^{N},x_{i}^{N})$$

where

$$x_t^N = (x_t, x_{t+1}, \dots, x_{t+N-1})$$

 $x_s^N = (x_s, x_{s+1}, \dots, x_{s+N-1})$

 x_t^N and x_s^N represent our 'N-histories'. $I_{\varepsilon}(x_t^N, x_s^N)$ is an indicator function that equals unity if $\|x_t^N - x_s^N\| < \varepsilon$ and zero otherwise, $\|\cdot\|$ is the sup-norm and $T_N = T - N + 1$. The correlation dimension of $\{x_t\}$ is given by the following expression:-

$$\nu = \lim_{\varepsilon \to 0} \frac{\log C_{N}(\varepsilon, T)}{\log(\varepsilon)}$$

⁶ Although there are several types of dimension, the most commonly used in the literature are capacity dimension, information dimension, correlation dimension, kth nearest-neighbour dimension and lyapunov dimension [see Parker and Chua (1989) for detailed descriptions of these fractal dimensions]. A point has dimension equal to unity, a line has dimension equal to two while a cube has dimension equal equal 3.

⁷ This integral can be written in other ways. See Parker and Chua (1989) and Bajo-Rubio (1992).

and measures the spatial correlation of the data. If the data is generated by a chaotic system then v the correlation dimension should be small.

Brock, Dechert and Scheinkman (1987) have devised a procedure based on the following sample correlation dimension of the N-vector.

$$C_{N}(\varepsilon,T) = \frac{2}{(T-N-1)(T-N)} \sum_{t < s} I_{\varepsilon}(x_{t}^{N}, x_{s}^{N})$$

$$I_{\varepsilon}(x_{t}^{N}, x_{s}^{N}) = \begin{cases} 1 & \text{if } ||x_{t}^{N} - x_{s}^{N}|| < \varepsilon \\ 0 & \text{if } ||x_{t}^{N} - x_{s}^{N}|| \ge \varepsilon \end{cases}$$

under the null that the $\{x_i\}$ are i.i.d.

$$C_N(\varepsilon,T) \to C_1(\varepsilon,T)^N$$
 as $T \to \infty$

and
$$\omega_{N}(\varepsilon, T) = \sqrt{\frac{T[C_{N}(\varepsilon, T) - C_{1}(\varepsilon, T)^{N}]}{\sigma_{n}(\varepsilon, T)}}$$

is asymptotically N(0,1). Rejection of the null is consistent with some type of nonlinear dependence in the series. This dependence however may be the result of nonlinear stochastic or nonlinear deterministic system.⁸

3.1.2 Sensitive Dependence on Initial Conditions

One of the additional features of a chaotic system is the strong dependence on initial conditions. In short, errors which affect the initial conditions tend to grow exponentially so that a small error has a dramatic effect on the forecasting ability of the system. The lyapunov exponent is one of the popular measures of sensitive dependence. This exponent

⁸ For instance Brockett et al (1988) and Hsieh (1989) have found strong nonlinear dependence in daily rates.

is used to characterise the wandering behaviour of chaotic orbits. In short, it defines how quickly orbits diverge in phase space.

A simple explanation of this exponent for a one-dimensional discrete dynamical system, $x_{k+1} = f(x_k)$ which has initial condition x_0 is provided. The lyapunov exponent $\lambda(x_0)$ for this system is defined as follows:

$$\lambda(x_0) = \lim_{n \to \infty} \frac{1}{n} \ln \prod_{k=1}^{n} |f'(x_k)|$$

The exponent depends on the initial condition and varies therefore for different orbits. For each value k, $f'(x_k)$ indicates how much the function is changing w.r.t its argument at the point $x = x_k$. The limit of the average of the log of the derivatives over n iterations provides a measure of how fast the orbit changes as the system propagates. If $\lambda(x_0) < 0$ then neighbouring orbits will remain close to each other whereas if $\lambda(x_0) > 0$ then orbits will diverge rapidly from nearby initial conditions divergence of orbits from nearby initial conditions. As such therefore a useful test for the existence of chaos is to determine the sign of the largest lyapunov exponent. The presence of at least one positive exponent can be taken as a sign of chaos.

Several algorithms have been developed in the literature to compute the largest exponent but the most popular is that of Wolf, Swift, Swinney and Vaston (1987)¹⁰. A useful program utilising this algorithm is contained in the work of Peters (1991).

⁹ As regards a fixed point in three dimensions, the lyapunov exponents are all negative. For a limit cycle, two exponents are negative while the third is zero and for chaotic behaviour, at least one of the exponents is positive.

¹⁰ Kurths and Herzel (1987) have also developed a useful algorithm.

Preliminary Results

The BDS statistic was computed for the daily buying rates of change of the three countries under investigation. Computations were largely based on software supplied by Dechert (1987) which accompanies Brock et al's (1993) much referenced book. The BDS statistic attempts to test the null hypothesis that a time series is generated by a sequence of iid random variables against the alternative that the series is generated by a nonlinear deterministic system. Table 4 presents preliminary results for the variables RLGYUSB, RLJJUSB and RLTTUSB. In executing the test one is prompted to supply values of epsilon ε and the embedding dimension, N. The table also reports in parentheses the standard errors of the BDS coefficients. Division of the coefficients by their standard errors allows one to perform tests based on the normal distribution.

A cursory glimpse of the results suggests that there is evidence of non-linear dependence in the daily buying rates for Guyana and Jamaica. No evidence of non-linear dependence was found for Trinidad and Tobago. It is however more difficult to make a definitive statement on the existence of low dimensional chaos from the BDS results. As a second check on our findings, we also computed lyapunov exponents for RLGYUSB, RLJJUSB and RLTTUSB. For the entire sample period under study (i.e. April 13, 1993-October 18, 1996) all the largest lyapunov exponents turned out to be negative indicating an absence of chaotic behaviour. For 1993, two of the lyapunov values were slightly positive but these had magnitudes that were much closer to zero indicating the existence of limit cycles. For the period January 02-October 18, 1996, no evidence of chaotic behaviour was found using the lyapunov exponent.

Section 4: Summary and Conclusion

The findings from our on-going research seem to suggest that the underlying mechanism driving the exchange rate is non-linear. However, we are not able to state categorically

whether or not small movements in exchange rates in any 'initial' period lead to the onset of chaotic behaviour. The testing mechanism for chaotic attractors is still at an embryonic stage and no detailed tests exist to distinguish between non-linear stochastic behaviour and non-linear deterministic chaos. Future research will focus in some detail on the GARCH methodology applied to distributions with heavy tails such as the Student-t. An attempt will also be made to utilize more efficient algorithms for the computation of correlation dimensions and the lyapunov exponent. Indeed, one of the major problems plaguing our investigation was the computation of both the lyapunov exponent and the correlation dimension. Given the size of our data sets, an average run of the program written by Peters (1991) could take as much as 4-5 hours before convergence is achieved. We have included a copy for your perusal and for experimentation.

Table 1

CHRONOLOGY OF EXCHANGE RATE REALIGNMENTS IN CARICOM /1973-1990/

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Table 2
Empirical Moments for Logarithmic Differences of the Buying and Selling rates (Individual Years, 1993, 1994, 1995, 1996)

-	(.	Individua <u>l</u> Yea	irs, 1995, 199	4, 1995, 1996	<u> </u>	
		April 13	-December 3	1, 1993		
-	RLGYUSB	RLGYUSS	RLTTUSB	RLTTUSS	RLJJUSB	RLJJUSS
Mean	100.0033	100.0033	100.0070	100.0041	100.0440	100.0440
Variance	0.0015	0.0123	0.0855	0.0061	0.1440	0.1520
Std Dev	0.03906	0.110	0.2924	0.0782	0.3794	: 0.3898
Skewness	0.0015	0.1692	0.7624	1.4748	0.5903	0.6297
Kurtosis	1.6585	103.4873	4.7601	7.5775	2.8829	3.1599
		January 0	2 - December	31, 1994		
Mean	100.0057	100,0057	100.0091	100.0087	100.0026	100 0026
Variance	1),6029	10.0015	1.5500	1.4918	0.01406	0.0136
Std Dev	0.0543	+ 0.0389	1.2450	1 2214	0.1186	1).1166
Skewness	0.1383	4.1563	0.9285	0.9881	0.8713	0.7663
Kurtosis	44.3571	38.8075	50.5439	53.0847	9.4352	9.2717
		January (02 - December	31, 1995	.:"	
Mean	99.9989	1 99 9988	100.0011	100 0018	100.0128	100.0129
Variance	0.00022	0.0012	0.0326	0.0123	0.0265	0.0567
Std Dev	0.0466	0.03405	0.1805	0.1108	0.1628	0.2382
Skewness	-0.7849	-1.3265	-0.2592	0.1746	4.6477	1.3888
Kurtosis	36.5224	24.2552	2.4882	2.8908	63.7710	72.3849
		January	02 - October	18, 1996		
Mean	100.0010	100 0010	100.0112	100.0096	99.9822	99.9823
Variance	0.0014	1 0.0022	0.0150	0.0069	0.0226	0.0249
Std Dev	0.03762	0.0470	0.1225	0.0831	0.1504	0.1577
Skewness	0.3445	1.3537	1.4699	6.5575	-0.3176	-0.3706
Kurtosis	3.1403	55,3027	6.4045	61.9507	11.3664	13.6550

Table 3
Empirical Moments for Logarithmic Differences of the Buying and Selling rates
(Full Sample Period, April 1993 - October 1996)

April 13 - October 18, 1996						
	REGYUSB	RLGYUSS	RLTTUSB	RLTTUSS	REIJUSB	RLJJUSS
Mean	100.0021	100.0021	100.0063	100.0056	100.0117	100.0117
Variance	0.0021	0.0039	0.4636	0.4219	0.04852	0.0606
Std Dev	0.460	0.0624	0.6809	0.6495	0.2203	0.2461
Skewness	0.0686	0.5529	1.5973	1.8385	1.5487	1.3902
Kurtosis	41.4635	232.8761	161.6037	189.8048	16.0721	28.1540

Table 4
Results based on the BDS Statistic

£	N	REGYUSB	REJJUSB	RLTTUSB
		APRIL 13, 1993 - 6) OCTOBER 18, 199	5
0.03	2	*0.058 (0.0017)	*0.057 (0.016)	0.003
0.03	3	0.015 (0.37)	0.15 (0.17)	0.001
0.03	4	*1.72 (0.62)	*2.31 (0.40)	0.19 (0.41)
		APRIL 13 - DEC	EMBER 31, 1993	
0.03	2	*1.21+ (0.07)	*1.211 (0.05)	(0.21 (0.35)
0.03	3	*2.60 (0.67)	*2.415 (0.056)	0.39 (0.42)
0.03	4	3.15 (0.002)	*2.90 (0.013)	0.79 (0.63)
		JANUARY 02 - O	CTOBER 18, 1996	
0.03	2	*1.386 (0.970)	*1.16 (0.09)	0.041
0.03	3	*1.75 (0.213)	*1.15 (0.21)	0.879 (0.915)
0.03	4	*1.96 (0.58)	*2.18 (0.93)	0.74 (0.613)

Notes: () figures in parentheses represent standard errors.

Results are based on the program supplied by Dechert (1987).

 $[\]ensuremath{^*}$ indicate significance at the 5% level.

Table 5
Sign of Highest Lyapunov Exponent
(Preliminary Results)

N	BGYUS	BJJUS	BTTUS
	APRIL 13, 199	3 - OCTOBER 18,	1996
2	(-)	(-)	(~)
3	(-)	(-)	(-)
4	(-)	(-)	(-)
	APRIL 13 - I	DECEMBER 31, 1	993
2	(+) approx. zero	(+)approx. zero	(-)
3	(-)	(-)	(-)
4	(-)	(-)	(-)
	JANUARY 02	- OCTOBER 18.	1996
2	(-)	(-)	(-)
3	(-)	(-)	(-)
4	(-)	(-)	(-)

Notes: Signs presented here are for the largest lyapunov coefficients based on the alogrith by Wolf et al (1988). Peters (1991) supplies a useful program in Basic.

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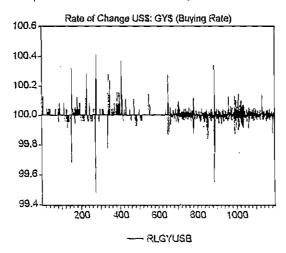
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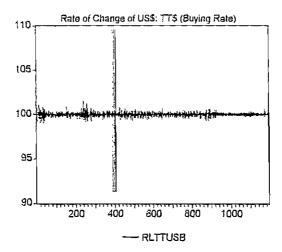
OBASIC PROGRAM TO COMPUTE THE CORRELATION DIMENSION (PETERS (1991))

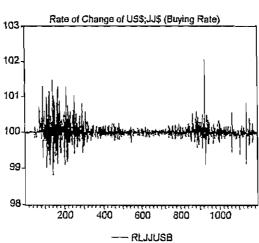
```
10 DIM X(1000)
20 DIM Z(1000, 10)
30 OPEN "C:\EXCHRAT\LYAP.PRN" FOR OUTPUT AS 2 LEN = 500
40 VT$ = "###,###### ##.#### ##.#####"
50 PRINT "INPUT NPT, DIM, TAU, DT, R"
60 INPUT "NUMBER OF OBSERVATIONS"; NPT
70 INPUT "EMBEDDING DIMENSION"; DIMEM
80 INPUT "LAG TIME FOR PHASE SPACE": TAU
90 INPUT "INCREMENTS TO DISTANCE"; DT
100 INPUT "INITIAL DISTNCE"; R
110 THETA = 0: THETA2 = 0: CR = 0: IND = 1
115 K = 1: LAG = 0: SUM = 0: ITS = 0
120 OPEN "C:\EXCHRAT\TTDAT.PRN" FOR INPUT AS 1 LEN = 2500
130 VT$ = "##.#### ##.####"
140 \text{ FOR } I = 1 \text{ TO NPT}
170 INPUT #1, X(I)
175 NEXT I
180 \text{ FOR I} = 1 \text{ TO NPT}
195 FOR J = 1 TO DIMEN
200 Z(I, J) = X(I + (J - 1) * TAU)
205 NEXT J
210 NEXT I
215 PRINT "DATA FORMATTED"
220 NPT = NPT - DIMEN * TAU
225 FOR K = 1 TO NPT
235 FOR I = 1 TO NPT
240 D = 0
245
    FOR J = 1 TO DIMEN
250 D = D + (Z(IND, J) - Z(I, J)) ^ 2
255 NEXT J
260 D = SQR(D)
270 IF D > R THEN THETA = 0 ELSE THETA2 = 1
280 THETA = THETA + THETA2
285 NEXT I
290 LAG = LAG + 1
300 NEXT K
310 CR = (1 / (NPT ^2)) * THETA
320 PRINT #2, VT$; CR, R
325 L = L + 1: IF L > 12 GOTO 350
330 R = R + DT
335 CR = 0: THETA = 0: THETA2 = 0: LAG = 0
340 GOTO 225
350 END
```

GRAPH 1

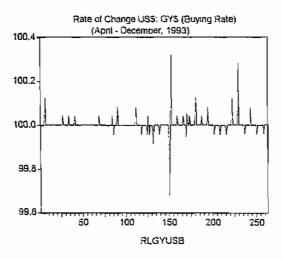
PLOTS OF EXCHANGE RATE CHANGES: (APRIL 1993 - OCTOBER 1996)

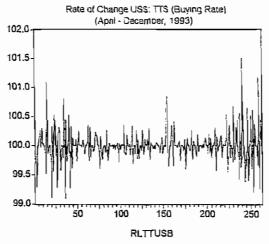


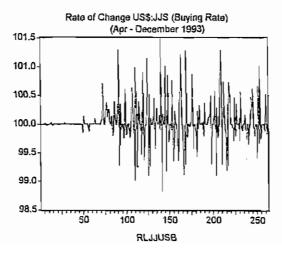




GRAPH 2
PLOTS OF EXCHANGE RATE CHANGES (APRIL 13 - DECEMBER 31, 1993)

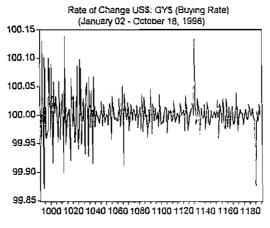




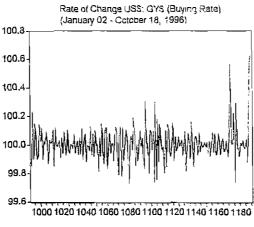


GRAPH 3

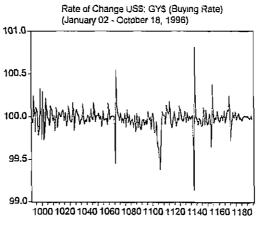
PLOT OF EXCHANGE RATE CHANGES
(JANUARY - OCTOBER, 1996)



--- RLGYUSB



---- RLTTUSB



--- RLJJUSB