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MODEL REFERENCE ADAPTIVE CONTROL OF THE SHORT RUN EXCHANGE RATE OF THE TNT DOLLAR

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model is non-linear, as most economic models. However, this approach linearises the model about operating points which are themselves time-varying.

Demand Side Dynamics

The demand for US$ from customers consists of three parts, the traditional demand related to price and governed by the typical demand/price curve, the speculation and need to preserve one's wealth which to a first approximation depend on the rate of change of the price, the liquidity of the TT$ in the country which affects both of the above demand curves.

Supply Side Dynamics

For a given supply of US$ to the country, the price of the US$ (the exchange rate) and a corresponding demand coming from customers, the banks will adjust their prices to reduce the imbalance between supply and demand. Hence the imbalance will be negatively related to the rate of change of the exchange rate. The feedback control by the Central Bank based on the rate of change of the exchange rate can be used either to put foreign exchange into the system or take it out. Since both the liquidity and to a certain extent the ability of the Central Bank to directly adjust the supply of foreign exchange in the market, this institution will have a significant effect on the control of the monetary environment.

Monetary Stability

This paper takes the strict mathematical definition of system stability. It does not include the popular impression of holding the exchange rate steady at some predetermined value (set point control in control jargon). This paper defines stability as the behaviour of the exchange rate in its own domain neighbourhood following a disturbance.

For the model described up to now the system is stable as long as there is no speculation. With speculation this model can remain stable if a particular relationship is maintained among the Central Bank's liquidity factor, the adjustment factor of the commercial banks and the speculation parameter. Clearly the system may be stable but its transient performance may not be as required. For instance due to sudden changes in the availability of foreign exchange the rate of change of the rate may not be beneficial to the economy or the business community. Thus, the criterion for control has to be more than stability in the sense of this paper.

Model Reference Control

The Central Bank cannot in the long run maintain a particular exchange rate by, for example, selling foreign exchange to the commercial banks if there is a sustained imbalance between demand and supply; the latter as earnings of the country. Thus level control by the Bank, the traditional interpretation of exchange rate stability, is not a feasible control policy objective. This paper suggests instead that the Bank may wish to propose a transient performance model which does two things:

i) relate the changes in the supply of US$ and the exchange rate at which there will be no need to buy or sell it FE to the market.

ii) define the rate of change of the exchange rate as it moves as a result of a system disturbance (eg. a change in the availability of foreign exchange to the market).

Even this can be modified in the light of control saturation. In this particular case this refers to when the Bank decides that it has no more US$ to sell and changes its reference model as the rate changes. In this case even the reference model becomes adaptive.

The problem can now be defined as:

> 'Select the Central Bank control parameters for liquidity control and the input/output of FE to the market to satisfy the Bank's model reference specification. The performance of the supply/demand system is non-linear but in this particular case it is modelled as a linear time varying parameter sub system."'

This is the classical Adaptive Control Problem.

Data

The crux of the control technique is the parameter estimation problem; i.e. one has to decide how the commercial banks behave, the normal customer demand as a function of the exchange rate and in particular an estimation of the phenomenon of speculation and the desire to maintain wealth. Also the Bank has to decide on its reference model.

If there were total freedom in how the Bank could intervene (for example, if the Bank could control the proportion of foreign exchange that hits the market and a feedback amount based on the current exchange rate) then there would be no need to directly estimate these parameters. The Bank would be exerting Direct Adaptive Control. This is the traditional state in which the Bank is a monopsony in foreign exchange.

However, the controls available to the Central Bank under this country's financial liberalisation are indirect and the result is as if the Bank were controlling the monetary system remotely. Hence some estimation of the parameters of the economic model is required.

This paper used stylised data simply to demonstrate that the algorithms work. Considerable difficulty was encountered in getting live data simply because they are not kept. The Bank admitted recently to this when it was stated that the demand for foreign exchange was not known (LLoyd Best, 'Floating Management, Drifting Leadership'). With the recent movement of the exchange rate the Bank will have to start tracking these parameters.

Inflows of Foreign Exchange

The model uses the available foreign exchange as an uncontrolled input. This is clearly a function of the economic performance of the country. The foreign exchange earned by the economy is a function of the production system. Hence the complete foreign exchange model is defined by the long term production system and the financial/monetary system. See figure 1, 2. Hence the complete model is made up of two loops, the outer or slow response loop involving the FE generated and required by industry which traditionally dominated the exchange rate literature.

Over the past few years the transactions that fall within the fast transient loop, particularly due to speculation and portfolio investment, are beginning to equal that of the traditional transactions in volume. Most of the published work deal with this slower loop but the financial liberalisation and the impact of computerised interconnected investment systems is making the inner loop assume greater prominence.

Future Work

This paper suggests improvement of the model as regards speculation modelling, system identification and the inclusion of other dependent variables (interest rates, inflation). However of particular importance is the use of real life performance information to determine the 'robustness' of the model and control algorithm. In this context robustness refers to the ability of the algorithm to handle the 'noise' associated with the measured variables and how good the dimension of the system model is compared to reality.

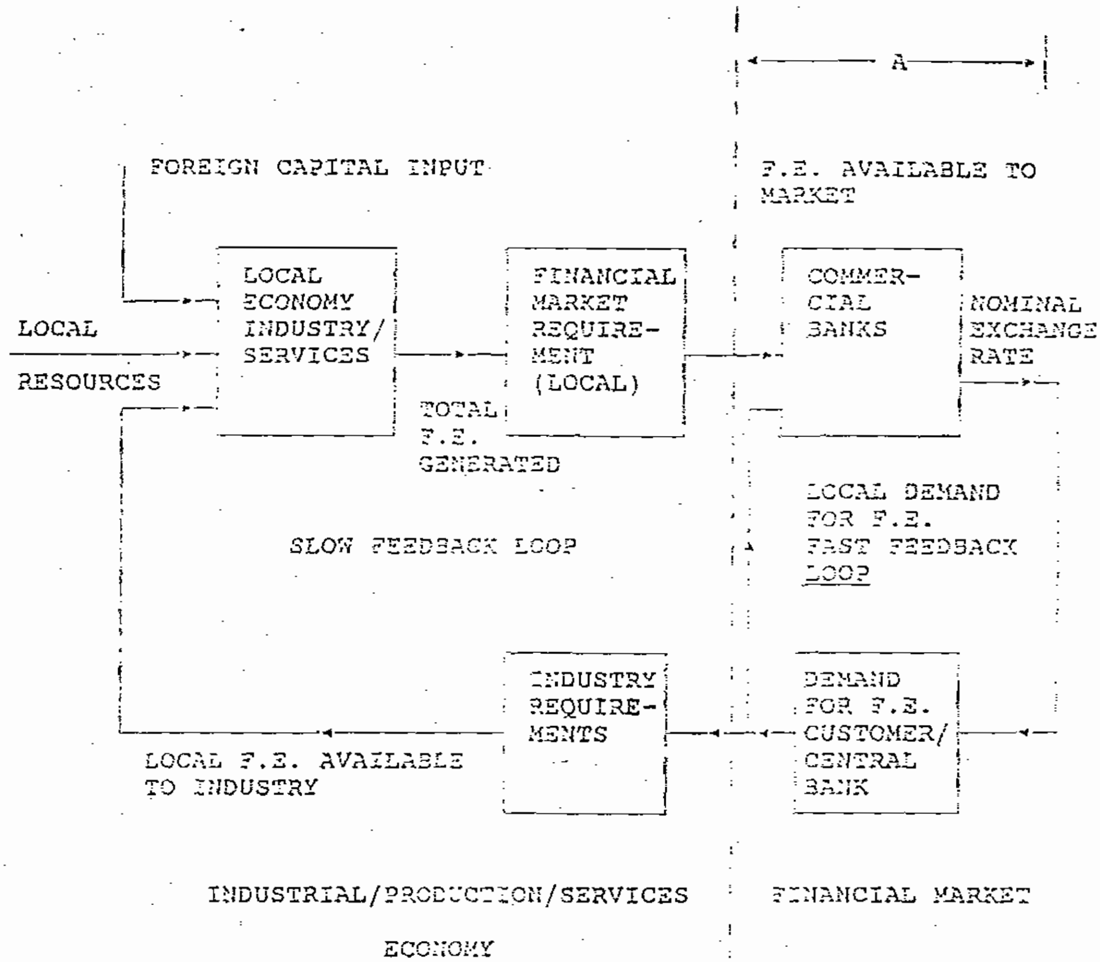
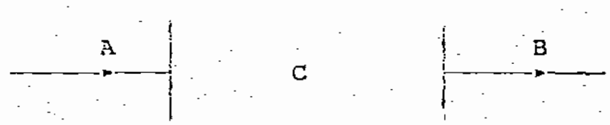


Fig. 1.1: SYSTEM DYNAMICS DIAGRAM (INPUT/OUTPUT)

KEY



Means input A via C causes output B

F.E. = Foreign Exchange

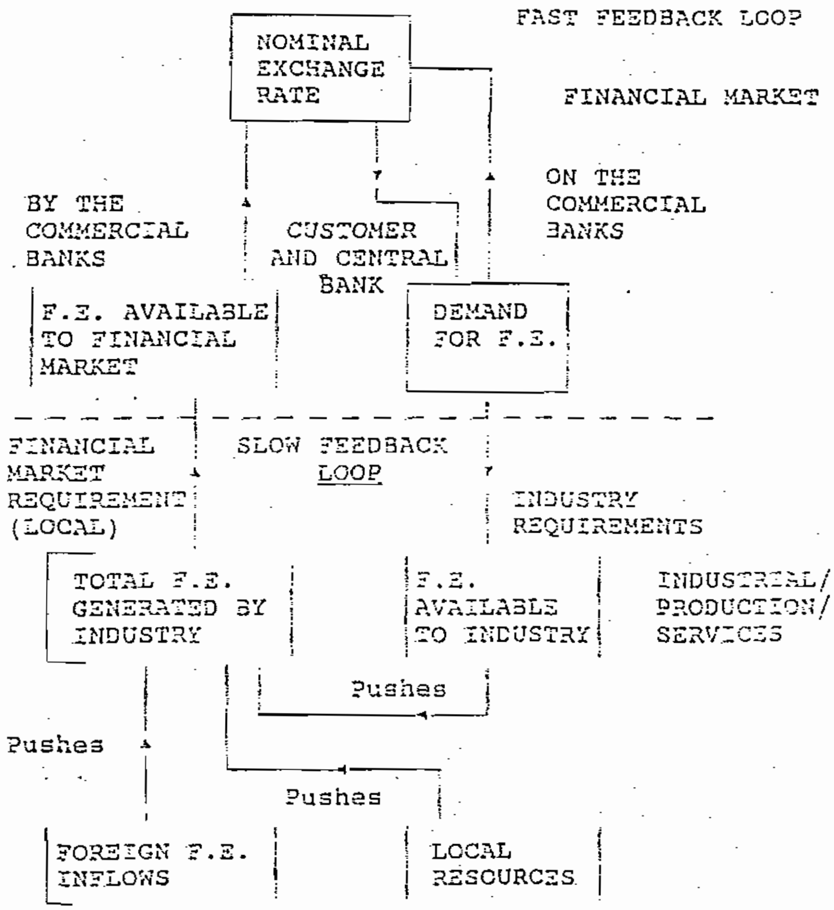


Fig. 1.2: EQUIVALENT DUAL CAUSAL DIAGRAM (SD)

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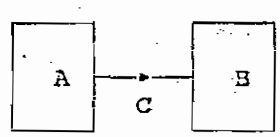

Means A via 'C' causes B

F.E. = Foreign Exchange

MODEL REFERENCE ADAPTIVE CONTROL OF THE SHORT RUN EXCHANGE RATE OF THE TRINIDAD & TOBAGO DOLLAR

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ABSTRACT

Most of the existing theories and literature on exchange rate control are based on long run[1] economic relationships. This paper introduces a modelling technique based on the short run relationships of the exchange rate, with particular reference to the role of the Central Bank in exercising control, and the effects of speculation. This was found to be an area of growing importance because of the significant number of short run transactions being conducted, with tremendous effect, singly or collectively, on exchange rate movements.

Model Reference Adaptive Continuous Time Theory is applied to the specific case of controlling the behaviour of the exchange rate of the Trinidad and Tobago dollar (TT$) with respect to the United States Dollar (US$) according to some predetermined dynamic performance criteria. The analysis is repeated for the discrete case (with allowance for input saturation as it felt that such a model more closely approaches reality). The results of applying test data to the computer programme developed for the model are then presented and discussed. Finally, the future direction of this research work is indicated.

INTRODUCTION

The overall behaviour of the financial system incorporating the exchange rate of the TT dollar (with respect to the US dollar) is determined by a fast transient (short-term) component and a steady state (long-term) component. The fast transient behaviour of the system is influenced by :

i) the elasticity of demand for US$ whereby demand is typically negatively related to its price;

ii) the availability of the TT$ in the local market, that is, its liquidity modelled in the large by a parameter that shifts the demand/price curve for the US$;

iii) speculation and the "need to preserve one's wealth" by customers; and

iv) the Central Bank which has the ability to sell and/or buy foreign exchange quickly in the market and also to control the liquidity of the TT$.

This model does not include the effect of bank interest rates since variations in interest rates ultimately affect liquidity.

This system when put together shows that the rate of exchange of the TT$ is a function of the available supply of the US$ with feedback effects due to customer demands (including speculation) and the Central Bank's involvement. It is this feedback that leads to the 'dual' or 'mutual causality' effect between the exchange rate and the demand for foreign exchange.

There is, however, a further feedback or dual causal relationship that connects the financial model, as described before, the economy (production of goods and services) ,the customer demand for the foreign exchange and its supply to the financial market - i.e. its inflows and outflows.

The inflows consist of :

- Foreign exchange input from the local market;
- New investments from foreign companies;
- Government revenue from the sale of oil, etc.;
- Divestment of local enterprises; and
- Exports from the local industry.

The outflows include :

- Debt repayments by the Government;
- Foreign purchases of raw materials and capital goods by the local industry.
- Domestic consumption (for travel, etc); and
- Capital flight

The overall foreign exchange pricing system, then, is made up of two loops - one defining the effect of the financial market (inner loop) and the other defining that of the production system (outer loop) on the price of the foreign currency. Fig. 1 is the System Dynamic diagram (Input/Output) linking both loops. Fig. 2 is the equivalent Dual Causal diagram for the system.

Traditionally, the outer loop (slow response) involving the foreign exchange generated and required by the industry dominated the factors affecting the exchange rate. The last few years, however, have shown that more and more of the transactions being conducted fall within the inner loop (fast cycle), particularly due to speculative forces and portfolio investment. Indeed, the total number of related transactions are probably now just about equally shared between both loops, demonstrating the growing importance of understanding the interaction of the supply and demand factors and their impact on price in the highly dynamic environment existing within the inner loop. Foreign currencies are now commodities to be traded in their own rights.

Fast Response Model

The supply of foreign exchange in the fast transient feedback loop is taken as a system input. Supply and demand conditions are examined separately and then linked together in a model using adaptive control techniques.

Let :

$S_p(t)$ be the supply of US$ to the local economy;

D(t) be the demand for US$ at any time "t"; and

P(t) be the price in TT$ of US$ at any time "t",

all measured about some operating point or psuedo steady state value.

The economic model is non-linear. A typical method of analysing such a model is to linearise about an 'operating' point giving the small signal model with its associated parameters. In the synthesis that follows, this is the method used except that the psuedo operating or equilibrium point is assumed to change (dynamic setting) and hence its changing 'parameters' will have to be continuously identified as the solution proceeds.

Demand Side Dynamics

The demand for US$ from customers is made up of 3 parts :

i) The traditional demand related to price governed by the typical demand-price curve and given by :

$D_1(t) \quad = \quad -K_1 P(t)$

where K_1 is the slope of the tangent to the demand curve at the operating point (one of the psuedo steady state parameters).

ii) The speculation or need to preserve wealth aspect which depends, as a first approximation, on the rate of change of price P(t), ignoring temporarily the other "prediction" inputs that affect P(t), and given by

$D_2(t) \quad = \quad K_2 \, dP/dt$ where k_2 is the speculation parameter.

iii) The liquidity (θ) of the TT$ in the country at time "t", which affects both the demand curves and the amount available for speculation.

From these 3 parts, the total consumer demand is given by:

$D(t) = \quad \theta[-K_1 P(t) + K_2 dP/dt]$

Supply Side Dynamics

As a first estimate, given a particular supply of US$ available to the Commercial Banks and Cambios, the price paid for the US$ and a corresponding demand coming from the customers, the banks will continually adjust their prices to reduce this imbalance between supply and demand. That is to say, any change in the price of the US$ is a direct function of this imbalance or shortage (assuming that the banks and cambios do not speculate).

This yields :

$$S_p(t) - D(t) = -(1/K_3)\ dP/dt$$

where K_3 is the respective bank factor influenced by internal policy at the time (-ve sign included because the price of the US$ decreases with an increase in supply over demand). The cascade effect of the earners of FE holding part of their earnings in the foreign currency and not making it available to the local banks can be modelled implicitly on the demand side of the equation.

Role of the Central Bank

Feedback control by the Central Bank based on the rate of change of the exchange rate can be used to either put money (US$) into the system or take it out (assuming the Central Bank can buy or release foreign exchange).

i.e. $\Delta Sp(t) = K_c\ dP/dt = S_c(t)$

Since both θ (liquidity) and K_c (Central Bank factor) are under the control of the Central Bank, this institution will have an impact on controlling monetary stability by its manipulation of θ and K_c. Using the restricted mathematical definition of Nyquist stability this model of the foreign exchange system can become unstable if

$$k_2 > 1/(k_3\theta)$$

i.e. the action of the speculator can drive the system unstable if the bank parameters are badly chosen. Note also that the system is always stable if there is no speculation. However, the Central Bank may want to have a more substantial effect on the behaviour of the local financial system. It may wish to force the system to behave according to preconceived ideas, i.e. according to dynamic performance criteria defined by a 'Reference Model'.

The Reference Model

A simple first order system of the form $K_n/(1 + \tau_n s)$ (Laplace Model) can be chosen to represent the reference model, where in practice K_n gives the desired ratio between changes in supply of US$ and the exchange rate, and the model parameter, τ_n, gives the desired rate of change of the exchange rate. Hence the Central Bank can specify both how quickly and by how much it wishes to see the exchange rate move for any monetary disturbance, a more sophisticated control specification than simply demanding Nyquist stability. Since K_1, K_2 K_3, K_n and τ_n are known or identifiable, it is possible to choose K_c and θ to force the economic model to behave as the Central Bank requires.

However, the parameters of the economic model change with time, hence K_c and θ will have to be continually changed. **This is the fundamental Adaptive Control problem in which K_c and θ must be changed to reflect changes in the economic parameters, yet ensure that this act of changing K_c and θ does not drive the system unstable.** The approach used is based on the Lyapunov method (for the continuous system) and Popov's Stability Criterion (for the discrete case)[3].

The system as described up to now is continuous and gives an adaptive non-linear solution that assumes that the Central Bank has access to the market on a minute by minute basis and that the parameters that define the economic model being used by the Bank are also so available.

In the real scenario the Central Bank will intervene in the market at discrete intervals; the Commercial Banks set their exchange rates at discrete intervals and the economic modelling is also discrete. Therefore, a more realistic though and more complex control mechanism must reflect the sampled data or discrete property of the money market. See Appendix 1. In fact if the bank parameters are badly chosen including the sample rate (the rate at which the banks get information from the market) the system can become unstable even without speculation.

A computer programme was developed from the analysis defined in Appendix 1 for the discrete adaptive model, which uses θ_n and $K_{c,n}$ as the controlling parameters.

Test data were applied to the programme and the results are discussed in Appendix 2.

Adaptation of the Reference Model

To extend the analysis one step further, consider what happens when the Central Bank cannot respond to the demand for foreign exchange; i.e. it either has none to sell or it prefers not to sell. In this case, the system becomes saturated and the Central Bank factor (K_{cn}) becomes zero. This means that only one parameter (θ) can now be chosen, making it impossible to specify a desired model as before.

In such a case, the performance criteria is the result of the adaptive adjustment of the reference model itself. That is, the Central Bank could choose to specify 'a', the response time. Thus, for improved performance, θ_n (the liquidity factor at t = nT) and K_n (the multiplication factor of the reference model at t = nT) now become the two parameters for the adaptive control of the system, i.e. the reference model itself now becomes adaptive.

Future Work

The model, as developed will be enhanced in several ways to reflect more accurately the real economic system.

(1) Prediction

Throughout the analysis, speculation and/or control of the system has been causal. Prediction by any of the players, particularly the speculator, has been based simply on the rate of change of the price of the US$. However, in reality, the speculator may look at other economic, political or social factors to predict the change in the US$ price. Prediction based on these factors (some of which are the price of oil, level of confidence in the economy, actual US$ reserves of the Central Bank) will be introduced into the model for improved performance.

(2) System Identification

In order to effect more accurate control on the system developed, it is necessary to continuously identify (online) the changing system parameters (K_1, K_2, K_3, etc). Such an identification system will be superimposed onto the model in 'real life' tests.

(3) Inclusion of additional variables

The primitive model of this paper used liquidity as an independent variable which in reality affects other variables eg. interest rates and inflation. These 'dependent' variables could be incorporated into a more detailed model to allow a clearer interface with the business world - high interest rates, for example, tend to decrease the liquidity also of the local currency. The model used lumps both the direct control of the Central Bank and the commercial bank's interest rates into one parameter, liquidity. They can be separated into two dependent parameters.

(4) The performance of the adaptive control system depends on the robustness of its design. This property which deals with the measurement noise of the system and its unmodelled dynamics will be investigated in later work.

There are some examples of foreign exchange regimes in the Caribbean region that showed violent domestic short term changes; Jamaica and Guyana with large depreciations. However, Trinidad and Tobago was able to maintain an almost constant level of price of foreign exchange over the past two years. The ideas developed in this work will be applied to these three (3) cases.

Conclusion

This paper defines an initial attempt to apply Adaptive Control Theory to the control of the fast transient loop of Foreign Exchange Rate. Though it was tested without an on-line parameter identification system and with highly stylised data the results demonstrate that the approach has interesting possibilities both from increasing the sophistication of the model and the choice of control parameters.

The dynamic performance criteria used in the design of the control moved away from the traditional approach of maintaining the exchange rate constant to the specification on both the response rate to the change in supply of US$ and the ratio at which the exchange rate to monetary disturbance will settle. This approach is given some support by Dr. T. Farrell (ex Deputy Governor of the Central Bank of Trinidad and

Tobago) in his statement:

> "stability does not mean fixity - that it will not move, but rather that it moves within an acceptable range".

The work of this paper takes this view a step further - foreign exchange rate control means achieving a dynamic performance of foreign exchange movements within patterns that are themselves adaptive.

APPENDIX 1

DISCRETE-TIME ANALYSIS OF THE ADAPTIVE CONTROL SYSTEM

Let

$P(n)$ be the exchange rate at instant $t = nT$

$S(n)$ be the expected supply of foreign exchange at $t = nT$

θ_n be the liquidity factor at $t = nT$

$K_{c,n}$ be the Central Bank injection factor at $t = nT$

Then $P(n+1)$ is the exchange rate at $t = (n+1)T$

$P_m(n+1)$ is the exchange rate as obtained from the desired model at $t = (n+1)T$

and $e(n+1)$ is the error between $P(n+1)$ and $P_m(n+1)$; i.e. $e(n+1) = P(n+1) - P_m(n+1)$

Now θ_n and $K_{c,n}$ are the control parameters that will be used to minimise the error between $P(n+1)$ and $P_m(n+1)$

θ_n and $K_{c,n}$ are clearly functions of the error $e(n+1)$ and are both constant during the interval from $t = nT$ to $t =$

(n+1)T.

The adaptive control model will use a discrete integrator given by

$$\frac{T z^{-1}}{1 - z^{-1}}$$

and a discrete differentiator given by

$$\frac{1 - z^{-1}}{T z^{-1}} \qquad \textit{(inverse of the integrator)}$$

in the z-transform domain.

In the time domain, the discrete integration behaviour of the commercial banks is modelled by

$$P(n+1) \quad = \quad K_i U(n) T + P(n) \qquad \text{(A1.1)}$$

Where U(n) is the algebraic sum of the supply of and demand for foreign exchange.

It is worth noting that the Commercial Banks may use a different 'integrator' model. However, this analysis will constrain the discrete 'differentiator' for modelling speculation and the Central Bank's performance to be as defined before.

The desired performance of the system can be modelled by :

$$\frac{K_n z^{-1} T}{1 - a z^{-1}}$$

where K_n and a can be chosen by the Central Bank planners as the corresponding discrete model's settling ratio and response speed.

This then gives :

$$P(z) = U(z) \frac{K_3 T z^{-1}}{1 - z^{-1}}$$

$$P(z) [1 - z^{-1}] = U(z) K_3 T z^{-1}$$

$$z P(z) = P(z) + K_3 T U(z)$$

In the time domain : $P(n+1) = P(n) + K_3 T u(n)$ (A1.2)

The desired model (reference model) gives :

$$zP_m(z) = a P_m(z) + K_n T R(z)$$

(A1.3)

In the time domain : $P_m(n+1) = a P_m(n) + K_n T r(n)$ (A1.4)

Now the economic model leads to the following equation :

$$U(z) = \frac{R_z}{1 + K_3 (K_{c,n} - K_2 \theta_n)} - \frac{K_1 \theta_n P(z)}{1 + K_3 (K_{c,n} - K_2 \theta_n)}$$

$$U(z) = g_0 R(z) - g_1 P(z)$$ (A1.5)

where

$$g_0 = \frac{1}{1 + K_3 (K_{c,n} - K_2 \theta_n)} \qquad g_1 = K_1 \theta_n g_0$$

i.e.

$$\theta_n = \frac{g_1}{K_1 g_0}$$

So

$$K_{c,n} = \frac{1}{g_0} \left[\frac{1}{K_3} + \frac{K_2 g_1}{K_1}\right] - \frac{1}{K_3}$$

In the time domain : $U(n) = g_0 r(n) - g_1 p(n)$

The question of whether the economic model can be forced to exactly match the desired model for real values of g_0 and g_1 will now be considered. For perfect matching:

$$\frac{K_n T z^{-1}}{1 - a z^{-1}} = \frac{g_0 \dfrac{K_3 T z^{-1}}{1 - z^{-1}}}{1 + g_1 K_3 T \dfrac{z^{-1}}{1 - z^{-1}}}$$

Equating coefficients : $K_n = g_0 K_3$ or $g_0 = \frac{K_n}{K_3}$

$$a = 1 - g_1 K_3 T$$

$$\text{or} \quad g_1 = \frac{1 - a}{K_3 T}$$

But $g_0 = \dfrac{1}{1+K_3(K_{2,n} - K_2 \theta_n)}$ and $g_1 = K_1 \theta_a g_0$

Thus

$$\frac{g_0}{g_1} = \frac{1}{K_1 \theta_a} = \frac{K_n}{K_3} x \frac{K_3 T}{1 - a}$$

and hence $\theta_a = \dfrac{(1 - a)}{K_1 K_n T}$

Where K_1, K_n and a are known or identifiable.

Thus, it is possible to match the desired model with the actual model by choosing K_2 and θ_a (or indirectly g_0 and g_1).

As in the continuous case it is required to develop a policy to choose K_2 and θ_a (g_0 and g_1) as the economic model changes yet ensure that the overall system remains stable.

Discrete Adaptive Control Policy

The actual solution will now be found.

The adaptive equations are :

$$P(n+1) = P(n) + K_3 T u(n)$$

$$u(n) = g_0 r(n) - g_1 p(n)$$

and

$$P_m(n+1) = aP_m(n) + K_n\, T\, r(n)$$

where

$$g_0(n) = \frac{1}{1 + K_3\ (K_{c,n} - K_2\ \theta_n)}$$

and

$$g_1(n) = \frac{K_1\ \theta_n}{1 + K_3\ (K_{c,n} - K_2\theta_n)}$$

The error equations are obtained from :

$$P(n+1) = P(n) + U(n)K_3T$$

and $$P_m(n+1) = aP_m(n) + K_nT\, r(n)$$

i.e. $$e(n+1) = P(n+1) - P_m(n+1)$$

$$= P(n) - aP_m(n) + U(n)\ K_3T - K_nTr(n)$$

But

$$U(n) = r(n)g_0 - g_1P(n)$$

$$\Rightarrow \quad e(n+1) = P(n) - aP_m(n) + K_3T[r(n)g_0 - g_1P(n)] - K_nT\ r(n)$$

$$= a\ e(n) + K_3T[r(n)\bar{g}_0 - P(n)\bar{g}_1] \qquad \text{(A1.6)}$$

where

$$\bar{g}_0 = (g_0 - \frac{K_n}{K_3}) \qquad \bar{g}_1 = (g_1 - \frac{1-a}{K_3T})$$

Taking the z transform of (A1.6) leads to :

$$E(z) = (\frac{z^{-1}}{1-az^{-1}})K_3T\ [r(z)\bar{g}_0 - P(n)\ \bar{g}_1]$$

$$= [\bar{r}(z)\ \bar{g}_0 - \bar{P}(z)\ \bar{g}_1]$$

$$\Rightarrow \quad e(n) = \phi^T(n)\ \omega(n) \qquad \text{(A1.7)}$$

where $\phi^T(n) = (g_0,\ g_1)$

and $\omega^T(n) = [r(n), -P(n)]$

and

$$\overline{r}(z) = \frac{r(z)K_3T}{z-a}$$

$$\overline{P}(z) = \frac{P(z)K_3T}{z-a}$$

i.e.

$$r(n+1) = a\ r(n) + K_3T\ r(n)$$

$$P(n+1) = a\ p(n) + K_3T\ P(n)$$

To satisfy Popov's criterion for stability, the update law is[2]

:

$$\phi(n) = \phi(n-1) - \Gamma e(n+1)\ \omega(n)$$

where Γ is chosen as

$$\begin{matrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{matrix}$$

In our particular case, the parameter update law becomes:

$$g_0(n) = g_0(n-1) - \gamma_1\ e(n+1)\ r(n)$$

$$g_1(n) = g_1(n-1) - \gamma_2 e(n+1)P(n)$$

This reduces to :

$$g_0(n) = g_0(n-1) - \gamma_1\ e(n+1)r(n) \qquad \text{(A1.8)}$$

$$g_1(n) = g_1(n-1) - \gamma_2\ e(n+1)P(n) \qquad \text{(A1.9)}$$

Now, equations (A1.8) and (A1.9) indicate that $e(n+1)$ is required to find $g_0(n)$ and $g_1(n)$. But we have seen before that $g_0(n)$ and $g_1(n)$ are required to find $e(n+1)$.

To solve this dilemma, the a priori/a posteriori correction is used[2], whereby the a priori error $e^0(K)$ is used in the analysis as an approximation of e(K). The a poster-iori error e(K) is then subsequently calculated from $e^0(K)$ using a correction factor as below :

$$e(K) = \phi^T(K)\ \omega(K)$$

$e^0(K)$ is calculated using the values of the parameters obtained one interval earlier, i.e. $\phi^T(K-1)$

Hence $\quad e^0(K) = \phi^T(K-1)\ \omega(K)$

By Popov :

$$e(K) = \phi^T(K)\ \omega(K) = [\phi^T(K-1) - \Gamma\ e(K)\ \omega^T(K)]\omega(K)$$

$$= \phi^T(K-1)\ \omega(K) - \omega^T(K)\ \ \Gamma\ e(K)\ \ \omega(K)$$

$$e(K) = \frac{e^0(K)}{1 + \omega^T(K)\ \Gamma\ \omega(K)}$$

For the particular problem on hand

$$e(n+1) = \frac{e^0(n+1)}{1 + [\overline{r}(n),\ \overline{p}(n)]\begin{bmatrix}\gamma_1 & 0\\ 0 & \gamma_2\end{bmatrix}\begin{bmatrix}\overline{r}(n)\\ \overline{p}(n)\end{bmatrix}}$$

$$= \frac{e^0(n+1)}{1 + \gamma_1^2 r^2(n) + \gamma_2^2\ \overline{p}^2(n)}$$

where

$$e^0(n+1) = a\ e(n) + K_3T[r(n)\ g_0(n-1) - P(n)\ g_1(n-1)]$$
$$- K_n\ T\ r(n) + P(n)(1-a)$$

So that both $e^0(n+1)$ and $e(n+1)$ can be easily calculated.

APPENDIX 2

Results of test data applied to computer programme

Programme Assumptions

The programme developed for the discrete case assumes that initial conditions are steady state about some operating point, i.e. the system is in equilibrium initially, where e(n), S(n), P(n) and $P_m(n)$ are the variations about the chosen 'operating point' at interval 'n'.

Hence :

e(0) - the initial error - is zero;

S(0) - the initial variation in supply of foreign exchange about the operating point is zero;

P(0) - the initial variation in exchange rate of the reference model - is zero.

$P_m(0)$ - the initial variation in exchange rate of the reference model - is zero.

K_1, K_2 and K_3 are input parameters to the programme which would normally be supplied by an external identification system. For the purposes of testing the programme, K_1, K_2 and K_3 are chosen and manually entered by the user.

Thus any value of S(n) entered in the programme represents a **change** in the available supply of US$, i.e. the excess (+ve) or deficit (-ve) of US$ supply over operating point supply.

Similarly the values of P(n+1) obtained from the programme represents changes in these exchange rates from their initial steady state values. Positive values for P(n+1) and $P_m(n+1)$ represent a depreciation of the local dollar and negative values represent an appreciation.

The table of results obtained from the programme for five different input conditions as well as a graphical display of one of the input conditions (Charts 1, 2 and 3) are presented below.

Discussion of Results

Input Condition 1

For S(n) = 10, this means that there is an increased supply of US$ over operating point supply. This causes a drop in the price of the US$ as shown by the negative values of P(n+1) and $P_m(n+1)$. The same holds for $0 < S(n) < +\infty$

Input Condition 2

The same values of K_1, K_2 and K_3 are maintained in the first case, except that now the supply S(n) is changed. As expected, the values of θ_n and $K_{s,n}$ remain largely unchanged, but the system stabilises at a different exchange rate.

Input Condition 3

For S(n) = -5, it is seen that for a negative change in the supply of US$ the exchange rate goes up, as shown by positive values for the exchange rates. The same holds for $-\infty < S(n) < 0$.

Input Condition 4

K_1, K_2 and K_3 are now changed while maintaining the same S(n) as in Condition 3, i.e. the parameters of the system change as would be indicated by the parameter identification module. Stability is now achieved at different values of θ_n and $K_{s,n}$. The steady state exchange rates, however, remain the same as in Condition 3, since the idea is for the real system to behave like the model.

Input Condition 5

Compared with Condition 3, both the values of K_1, K_2, and K_3 as well as the value of S(n) are changed. In this case, it is shown that the steady state values of the parameters θ_n, $K_{2,n}$, P(n+1) and $P_m(n+1)$ are all different from those obtained in Condition 3.

Analysis of Graphics

Chart 1 is a representation of the movement of the exchange rate with time, for Input Condition 1, as pre-determined in the Reference Model, i.e. this is how the planners at the Central Bank would want the system to perform, given these particular values for the input variables {S(n) = 10, K_1=1, K_2=1 and K_3=1}.

Chart 2 is a display of how the actual exchange rate of the system moved with time over the same period, also for Input Condition 1, i.e. it showed the performance of the model developed to represent the financial system under the same values for the input variables outlined before.

The almost exact similarity of the two graphs shows that the Model Reference Adaptive Control system developed in this paper is achieving its intended purpose of stability of the exchange rate, insofar that the exchange rate of the actual financial system is being brought to an equilibrium in an almost similar manner to that required by the authorities.

Chart 3 is an exploded view of the two graphs for n = 1 to 10, showing that equilibrium is largely achieved over the first 10 iterations of the programme.

References

1. Engle R. F., Granger C. W., 1991; *Long Run Economic Relationships. Oxford Clarendon Press, pp 8-13, 82-85*
2. Butler H. *Model Reference Adaptive Control, from Theory to Practice. Prentice Hall pp 34-42, 110-115, 215*

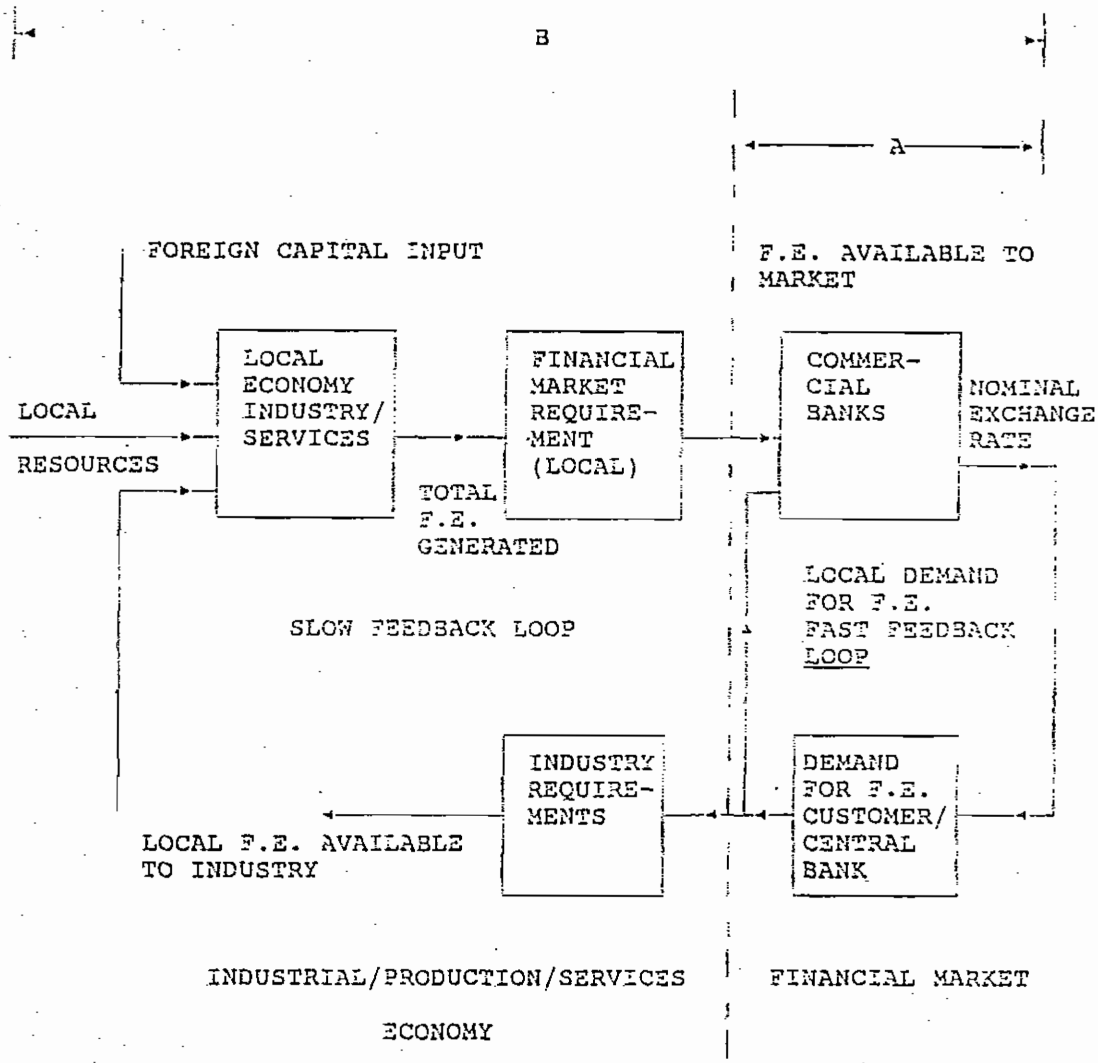
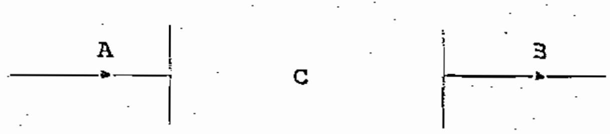


Fig. 1 : SYSTEM DYNAMICS DIAGRAM (INPUT/OUTPUT)

KEY



Means input A via C causes output B

F.E. = Foreign Exchange

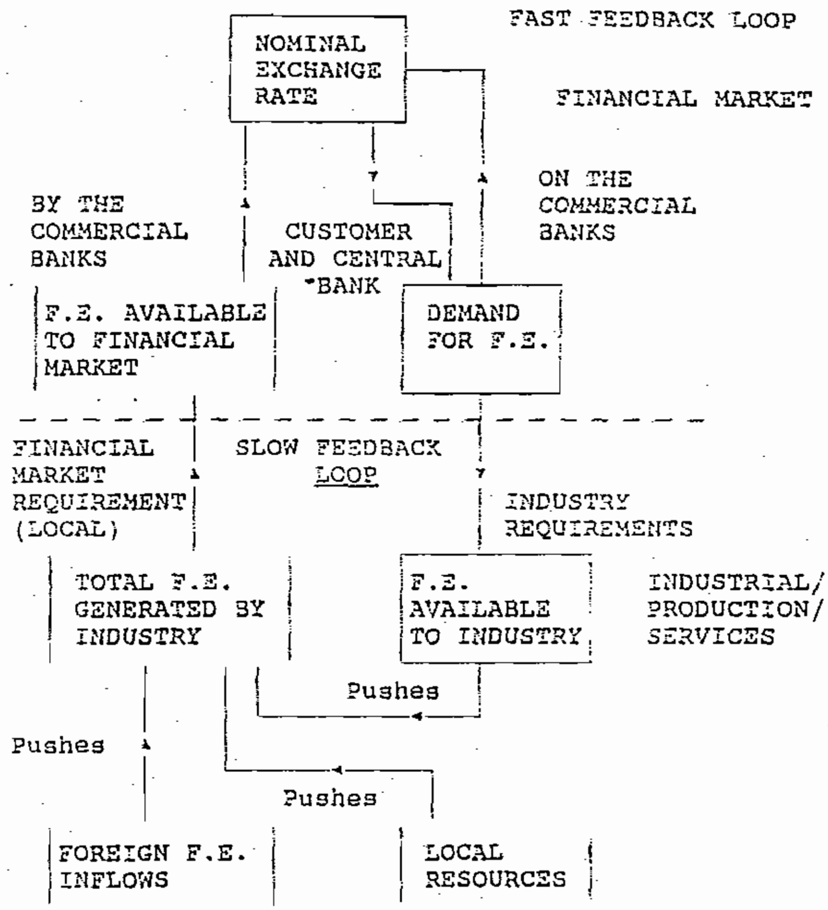


Fig. 2: EQUIVALENT DUAL CAUSAL DIAGRAM (SD)

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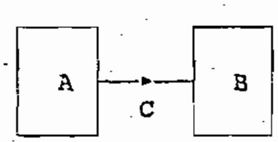


Means A via 'C' causes B

F.E. = Foreign Exchange

Programme Results

Using a = 0.5, Kn = 15, T = 0.01, γ_1 and γ_2 = 1, the results for five different input conditions are shown below:

Input Condition	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1	1	1	1	10	3.333333	2.400000	-3.000002	-3.000000	-0.000002	3.333331	2.400667
2	1	1	1	15	3.333333	2.400000	-4.500001	-4.500000	-0.000001	3.333333	2.400710
3	1	1	1	-5	3.333333	2.400000	1.499995	1.500000	-0.000005	3.333344	2.400495
4	2	3	4	-5	1.666667	4.816667	1.500000	1.500000	0.000000	1.666667	4.820326
5	2	3	4	-20	1.666667	4.816667	5.000000	5.000000	0.000000	1.666667	4.818058

Chart 1

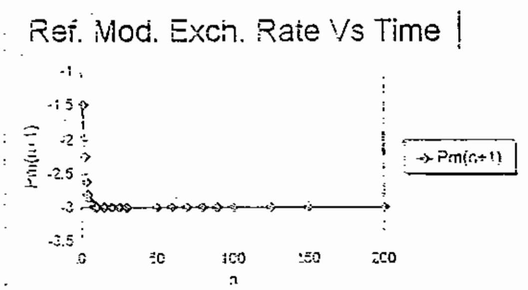


Chart 2

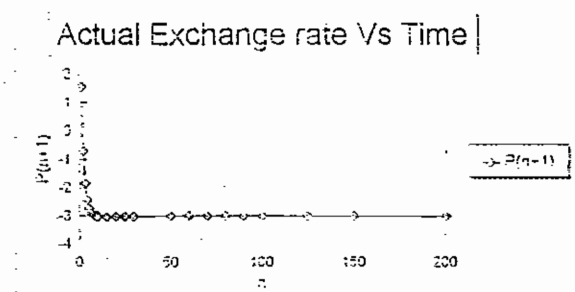
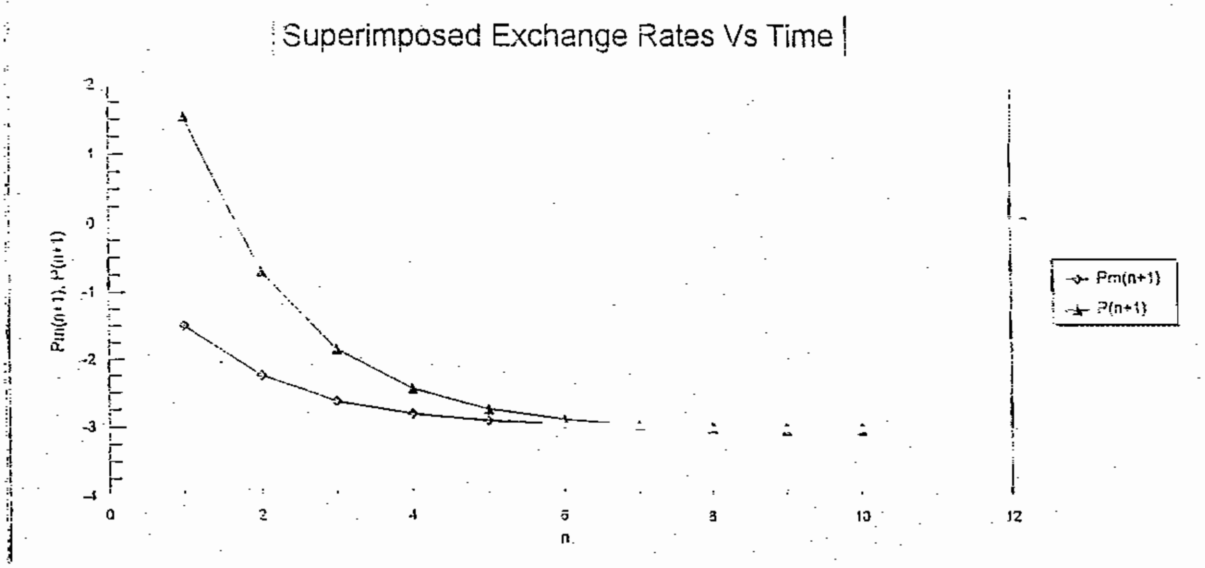


Chart 3



Appendix 3

Review of Existing Theories on Exchange Rate Control

Monetary Approach to Balance of Payments (MABP)

Consider the balance sheet of the consolidated monetary sector (i.e. the aggregation of the Central Bank, the commercial banks and all the issuers of money) which contains Assets (D,R) and Liabilities (M), where M is money supply, r is the foreign based component of M (foreign reserves) and D id the domestic or local assets of the banking system (giving $M=R+D$).

The MABP proposes that for a fixed exchange rate system and assuming full employment a change in the domestic component of the money supply will lead to a largely offsetting change in reserves, resulting in a constant overall money supply and therefore a constant exchange rate.

With a floating exchange rate by definition the authorities do not intervene in the foreign exchange market and thus the change in reserves must always be equal to zero. Hence any change in D is matched by a change in M, leading to a change in the price of foreign exchange.

Purchasing Power Parity (PPP)

With fixed rates the PPP relationship purports that a country's inflation rate cannot deviate from the world's rate. Under floating exchange rates PPP becomes a theory of the determination of the exchange rate as shown below.

Consider a good, 'i', which is produced in the USA and the UK and is freely tradeable between the two countries. If Pi.uk is the price of the good in the UK and Pi,us is the US price, then the following 'law of price' must hold:

$P_{i,uk} = P_{i,us} \times S$

where S is the exchange rate between the Pound and the dollar. This equation can be expanded for a range of goods which lead to the obvious definition of the exchange rate. The difficulties associated with both versions (fixed and floating) of the PPP are:

i) In the definition of the PPP only traded goods prices are included. However all countries provide a range of services which are non traded and enter the computation of price indices. On the other hand if the price index which include both trade and non-traded goods and services is used, productivity differences between countries can impart a bias into the calculation od PPP.

ii) PPP is not likely to hold continuously as a theory for the determination of the exchange rate, because a number of factors including trading restriction and internationally mobile capital push the actual rate away from the PPP determined rate.

PPP is not well supported by empirical evidence and is at best only a partial theory of rate determination.

Balance of Payments (Flow) Model

This model treats the exchange rate as a price and like any other price uses supply and demand in order to analyse its determination.

From the stability point of view any change in the exchange rate from equilibrium should produce a corresponding supply change which then moves the exchange rate accordingly.

Although this model caters for both long and short term transactions it does not adequately address the non-linearity of the problem nor the dynamics of the system.

Econometric Methods

Asset Market Models

Asset market models concentrate on the stocks of money and bonds outstanding. These in turn depend upon the level of interest rates, adjusted for expected changes in the exchange rate. The equilibrium exchange rate is determined by the level of asses stocks in excess (or short) of demand. Asset market models of exchange rate determination are expressed in terms of four basic relationships involving the variables: nominal money supply, price level, real income and nominal rate of interest. These relationships when solved lead to an equation for the determination of the exchange rate. The resulting equation is not a genuine reduced-form equation (since it contains endogenous variables among the explanatory variables) and is neither static equation nor a properly specified dynamic equation.

Dynamic Time Series Models (Error Correction and Co-integration)

Some economic variables can be viewed as a time series. Pairs of such series can either vary extensively from each other or can be expected to move so that they are not too far apart. This forms the basis of a class of models known as 'error correcting' which allows long run components of variables to obey equilibrium constraints while short run components have a flexible dynamic specification. Generally for a two variable system a typical error correction model would relate the change in one variable to past equilibrium errors as well as to past changes in both variables. A condition for this to be true (called co-integration) was introduced by Granger in 1982 and Granger and Weiss in 1983. Of interest is that tests conducted by several researchers applied to the case of asset market approach led to the rejection of the assumed long run proportionality of the exchange rate and relative money supplies. These results considerably discount the validity of the asset market approach to the exchange rate. More specifically they suggest that the underlying long-run equilibrium condition ia an inappropriate theory upon which to base exchange rate theory.

Appendix 4

Adaptive Control

The two terms 'adaptive' and 'control' require some discussion. Taking the latter first, control system theory was developed to address the problem of controlling the outputs of a dynamic system based on some desired values and subject to some system specifications. In classical control of time invariant linear systems, the performance specifications are usually a transient and steady state response criterion. This is sufficient for many engineering type systems.

However, more stringent system specifications have been used and in particular in optimum control theory it is usual to require the system to 'move' from one state to another and in so doing optimise some cost function. For example, the cost function may be minimum time or fuel for a rocket system and yet another in economics may be the arrival time of an economy from its present state to another defined by certain parameters.

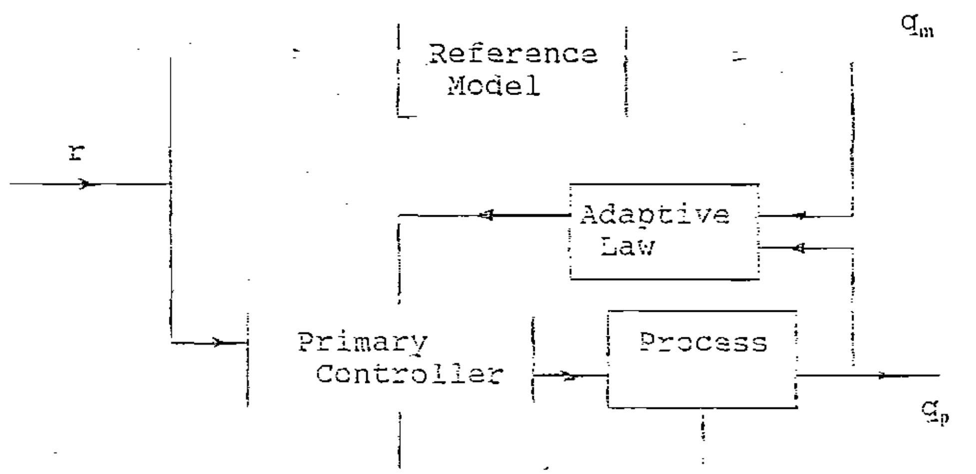
In the process of the development of modern control theory it was seen that a fixed control algorithm could not give acceptable behaviour for all conditions, particularly if the process to be controlled had unknown or time varying parameters. Hence in the late 1950's this led to an interest in adaptive control algorithms which could be applied if the process behaviour depended in a known way on some external and measurable condition.

In this particular case the foreign exchange rate of the TT$ (assuming that the short term supply to be an uncontrolled input) is a function of the commercial and Central Bank's responses to the difference between supply and demand for foreign currency. The parameters that define these responses are indeed time varying and can be measured or estimated from peoples' past behaviour. Control inputs by the Central Bank can then attempt to adjust this rate. Further , the dynamic models od the system are themselves non-linear. One method of addressing this problem is to linearise the model about the operating point and consider the operating point itself to be a time varying and measurable parameter.

A possible definition of an Adaptive Control System is one that adapts itself to changes in the process. What is important to appreciate is that continuous design of the controller parameters depends on being able to estimate the process parameters. Hence an adaptive control procedure always requires an associated estimation algorithm. If the estimation procedure first gives the parameters of the process then the method is termed 'indirect' adaptive control. It is possible to adjust the estimation procedure such that it directly gives the control parameter. This latter case is called 'direct' adaptive control and is usually easier to analyse. It is worth noting that the estimation scheme will add extra dynamics to the control scheme. These dynamics are always non-linear and hence complicates the analysis problem. The method used in this paper. MRAC, is a typical example of direct adaptation.

The MRAC was first introduced by Whitacker in 1958 and its most popular form is where a primary controller is used to obtain suitable closed loop behaviour, in a non adaptive way.

However, the process parameters are unknown or may vary with time. A fixed parameter setting for the primary controller such that the loop behaviour is acceptable under all circumstances cannot be found. In the MRAC the desired response to a command signal is specified by a parametrically defined model. An adaptation mechanism keeps track of the process output and the model output and calculates a suitable parameter setting such that the difference between them tends to zero.



MRAC Scheme

It has already been stated that the system is always non linear because of the adaptive scheme. In this case the process is time varying. As in the design of all control systems the transient stability of the design is of paramount importance. For a linear system the simple Routh Hurwitz criterion or its analogue in the discrete case, Shur Cohn, suffice. However, in the non linear and time varying case linear stability tools are of little use. To guarantee overall stability of the system the well known Lyapunov direct method can be utilised. Instead the Popov hyperstability approach can be used in which the designer has to propose and adaptive law and with the aid of hyperstability theory the stability is checked.

The performance of the adaptive control system depends on the robustness of its design. This analysis considers the boundness of all signals in the adaptive loop and is associated with both external bounded disturbances and state dependent disturbances. The first includes system and measurement noise and the second in which the process has unmodelled dynamics. The robustness of the foreign exchange system is not considered in this presentation. It is the subject of continuing work.