TIME-VARYING VOLATILITY: A MULTIVARIATE APPROACH

by

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Abstract: A vector error correction process with a multivariate GARCH variance structure is used to model stock returns on the Jamaica Stock Exchange. Structural restrictions are used to identify the estimated cointegration space. The results indicate long run relationships between the stock price and the treasury bill rate, and among traded volume, stock price and the real exchange rate.

Outline

This paper summarizes a procedure for analyzing relationships between stock returns and other macroeconomic variables. Typical estimates of volatility assume that the appropriate conditioning set for the mean function is a low order auto regression. On the other hand, the variance function has been augmented to include the effects of mixing variables like traded volume. Karpoff (1987) suggests that volume may be simultaneously determined with the price index of equities. To the extent that volume is an endogenous variable, its treatment as a pre-determined or exogenous variable can generate estimation biases.

The approach followed here assumes that there exists a vector of endogenous variables that exhibits common long run tendencies. In particular, we argue that the return on common stock is related to the return on short term treasury bills (term structure), the rate of inflation and changes in the exchange rate and traded volume. We assume that the levels potentially contain common trends that may not appear in short run dynamic influences.

The appropriate methodology for analyzing such a system is the Vector Error Correction Model (VECM). We consider the system

$$Z_t = A_1 Z_{t-1} + \ldots\ldots + A_k Z_{t-k} + \mu + \varepsilon_t, \qquad t = 1, \ldots\ldots T$$

where Zt is a p x 1 vector of stochastic variables with the first k observations treated as fixed. We can reparameterize the system as a VECM (Johansen (1988))

$$DZ_t = B_1 DZ_{t-1} + \dots\dots\dots + B_k DZ_{t-k+1} + \alpha\beta' Z_{t-1} + \mu + \varepsilon_t$$

where α and β are p x r matrices.

The first decision parameter relates to the choice of lag length of the Vector Autoregression (VAR) and the choice of deterministic components in the system. In our case, graphical plots and autocorrelation plots suggest that extreme classifications of no deterministic components or quadratic trends in the data can be eliminated from further analysis. The lag length chosen was the minimum lag that generated a system error structure that did not exhibit serial correlation. For the monthly Jamaican series of interest, five lags are sufficient. Conditional on that maximum lag length, the system is tested for the number of common trends or cointegrating relationships that exist among the five variables. The basic principle employed follows Pantula (1989) whereby alternative models under consideration are sequentially tested starting from the most restricted model and lowest cointegrating rank. In addition, tests for exclusion of individual variables, tests for stationarity of individual variables conditional on an estimated cointegrated space, and tests for weak exogeneity of individual variables are conducted as preliminary indications of systemic structure.

Our preliminary tests suggested a system with at most two cointegrating relationships or three common trends in a model that contains linear trends in the data and an intercept for the cointegrating relationships. The system was then estimated using the Johansen procedure to identify both the cointegrating vectors and the short run dynamic parameters. Given that the cointegrating space is not uniquely determined we proceed through the imposition of subset restrictions on the cointegrating relations to identify structural relationships within that cointegrated space. The matrix of cointegrating vectors and adjustment factors is shown below:

BETA

LSP	LTB	LVOL	LCP	LXH
-1.99	6.77	0.66	-0.40	0.38
-0.85	-1.15	0.92	-3.05	3.87
0.29	1.72	0.56	3.82	-6.12
-1.15	-0.94	0.10	1.55	-0.02
-1.02	-2.25	0.06	3.30	-0.18

ALPHA

LSP	LTB	LVOL	LCP	LXH
0.02	-0.01	-0.003	0.001	0.002
-0.015	-0.002	-0.002	0.002	0.002
-0.07	-0.18	-0.06	-0.05	0.002
0.001	0.002	0.00	-0.001	0.00
-0.002	-0.006	0.01	-0.001	0.001

Normalizing the two cointegrating vectors, our reduced alpha matrix has the following t-values:

T-VALUES FOR ALPHA MATRIX

DLSP	-4.45	2.68
DLTB	4.69	0.68
DLVOL	1.37	3.46

DLCP	-0.99	-2.63
DLXH	0.57	1.95

The beta and alpha matrices suggested hypotheses on structural relationships and weak exogeneity for the system. In particular, the likelihood ratio test failed to reject (p-value = 0.63) the null hypothesis of weak exogeneity for LXH. Conditional on LXH being weakly exogenous, the re-estimated partial system did not reject (p-value = 0.81) the structural hypotheses indicated in the beta matrix below:

BETA

LSP	LTB	LVOL	LCP	LXH
1.00	-4.58	0.00	0.00	0.00
-0.78	0.00	1.00	-1.286	1.286

T-VALUES FOR ALPHA MATRIX

DLSP	-4.66	0.03
DLTB	4.58	-3.08
DLVOL	1.06	-3.80
DLCP	-0.96	2.96

The corresponding t-values for the alpha matrix indicate that the first cointegrating vector which links the stock return with changes in the treasury bill rate enters both the stock return and treasury bill error correction equations. The second cointegrating relationship, which posits positive relationships between volume traded and both stock price and the real exchange rate, enters the treasury bill, volume and inflation error correcting equations.

The short run matrices indicate the following structure:

$$DLSP = f(DLSP_{t-1}, DLTB_{t-2}, DLTB_{t-3}, DLTB_{t-4})$$

$$DLTB = f(DLSP_{t-1}, DLTB_{t-1}, DLVOL_{t-1}, DLCP_{t-1}, DLTB_{t-3}, DLTB_{t-4})$$

$$DLVOL = f(DLSP_{t-1}, DLVOL_{t-1}, DLVOL_{t-2}, DLVOL_{t-3})$$

$$DLCP = f(DLCP_{t-1}, DLTB_{t-3}, DLXH, DLXH_{t-1}, DLXH_{t-2})$$

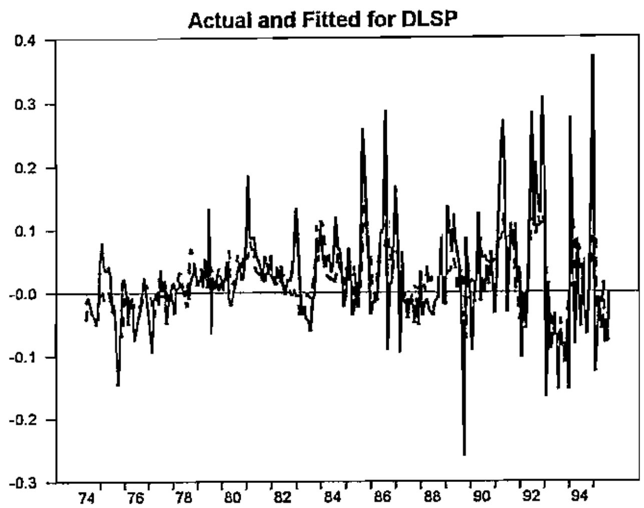
The correlation matrix of the residuals suggest a diagonal form:

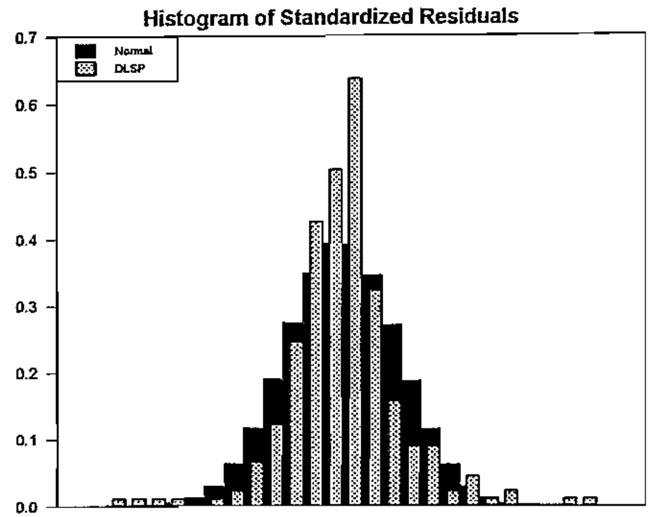
RESIDUAL CORRELATION MATRIX

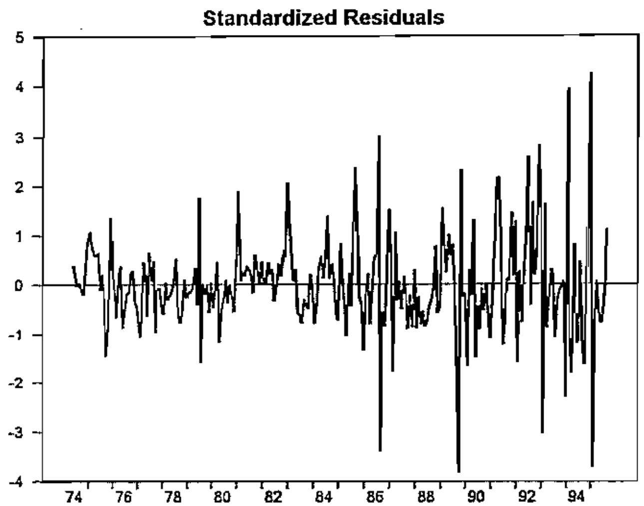
DLSP	DLTB	DLVOL	DLCP
1.00			
-0.03	1.00		
0.12	-0.02	1.00	
0.01	-0.16	0.04	1.00

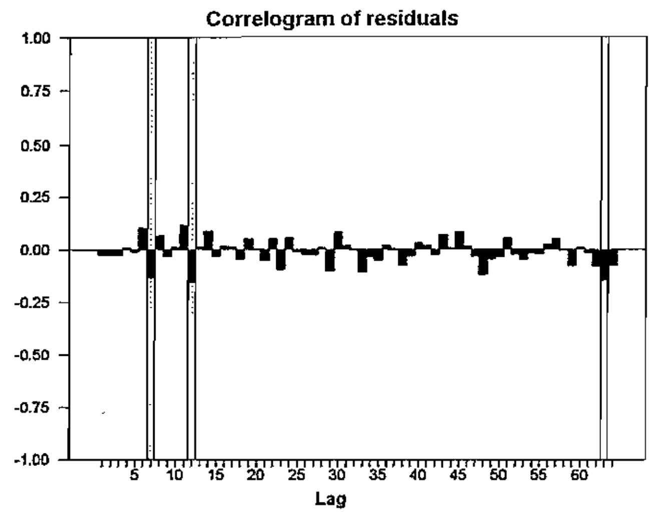
The system tests for the null hypothesis of no autocorrelation are not rejected for one (p-value = 0.44) and four lags (p-value = 0.93). The individual equation tests for normality and a system counterpart rejects the null hypothesis. In each case there is excess kurtosis and the stock return and treasury bill equations both fail ARCH tests for heteroscedasticity. The eigenvalues of the companion matrix indicate three values near the unit circle supporting the hypothesis of two cointegrating vectors. In addition, plots of the cointegrating relations indicate stationarity and a plot of the log likelihood values lies within the two standard error bands, suggesting constancy.

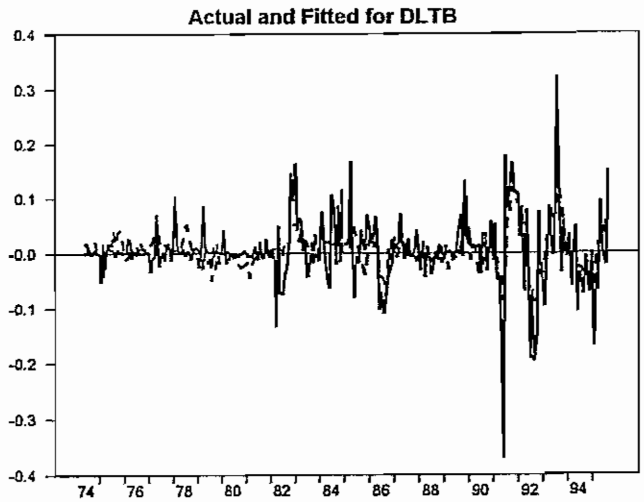
Given the above residual diagnostics, the identified structural system is estimated with a multivariate GARCH structure with a diagonal covariance matrix imposed a priori.

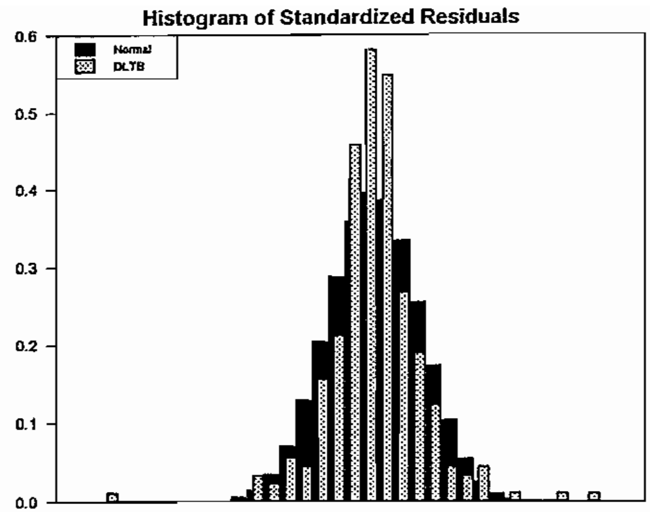

Actual and Fitted for DLSP
0.4
0.3
0.2
0.1
-0.0
-0.1
-0.2
-0.3
74
76
78
80
82
84
86
88
90
92
94

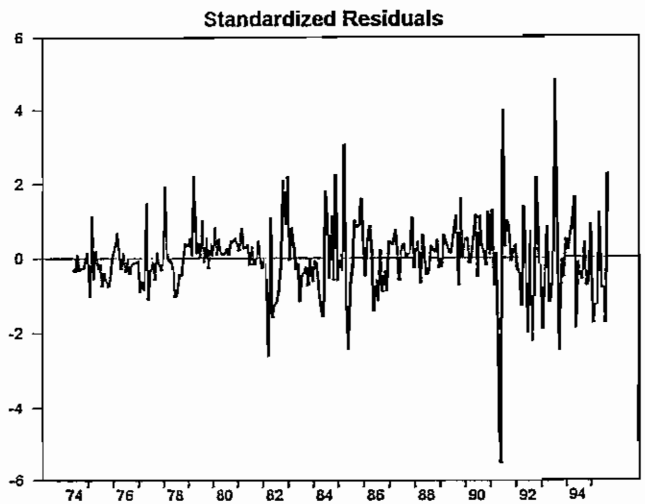

Histogram of Standardized Residuals
Normal
DLSP
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0

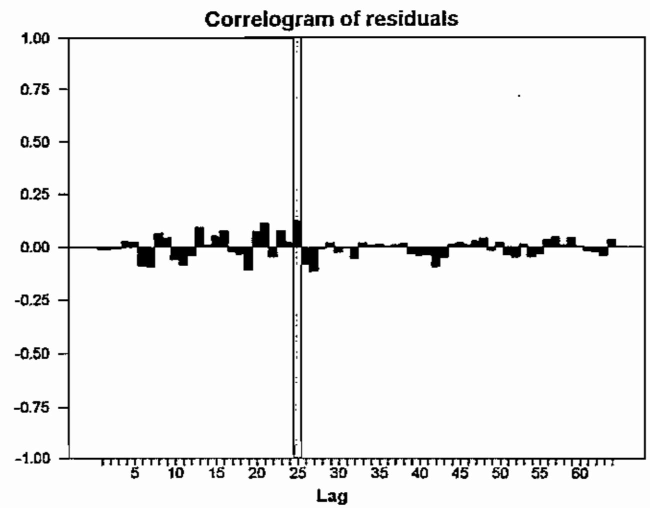

Standardized Residuals
5
4
3
2
1
0
-1
-2
-3
-4
74
76
78
80
82
84
86
88
90
92
94

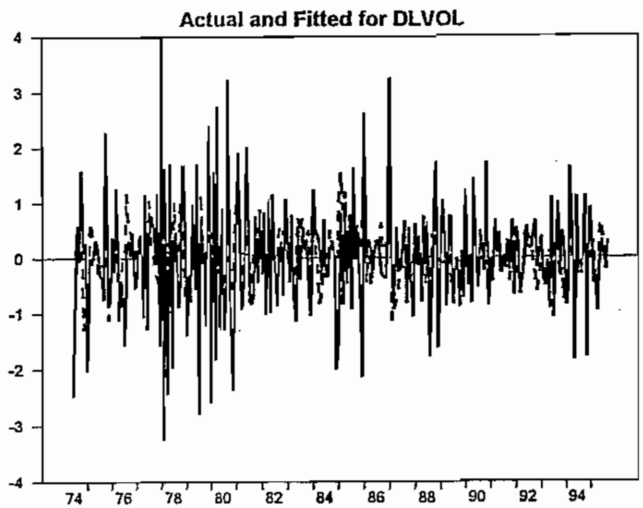

Correlogram of residuals
1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
-1.00
5
10
15
20
25
30
35
40
45
50
55
60
Lag

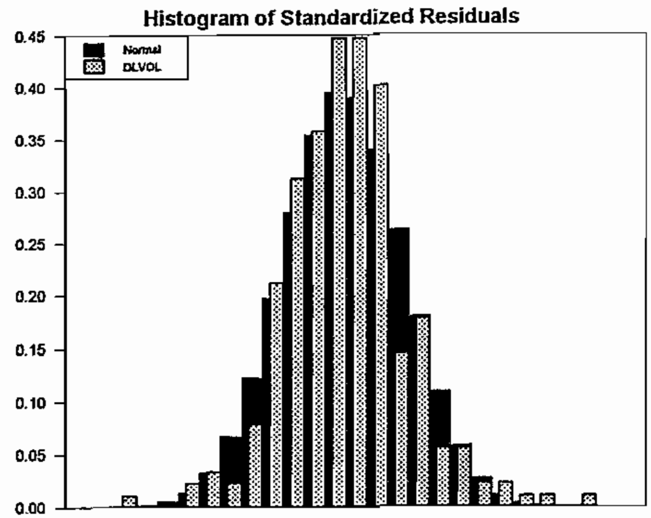

Actual and Fitted for DLTB
0.4
0.3
0.2
0.1
-0.0
-0.1
-0.2
-0.3
-0.4
74 76 78 80 82 84 86 88 90 92 94

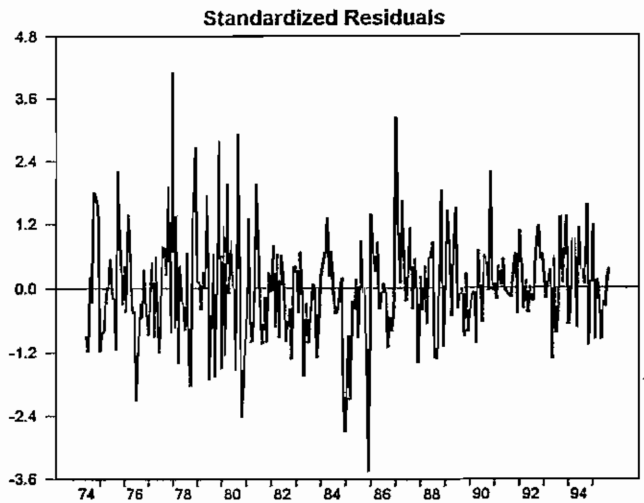

Histogram of Standardized Residuals
Normal
DLTB
0.6
0.5
0.4
0.3
0.2
0.1
0.0

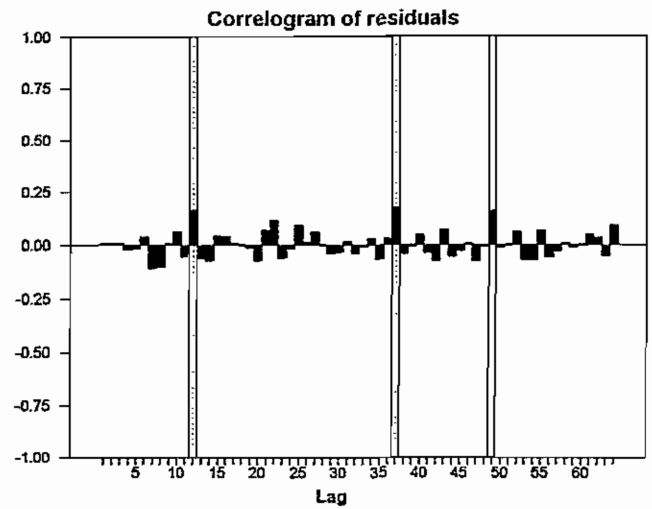

Standardized Residuals
6
4
2
0
-2
-4
-6
74 76 78 80 82 84 86 88 90 92 94

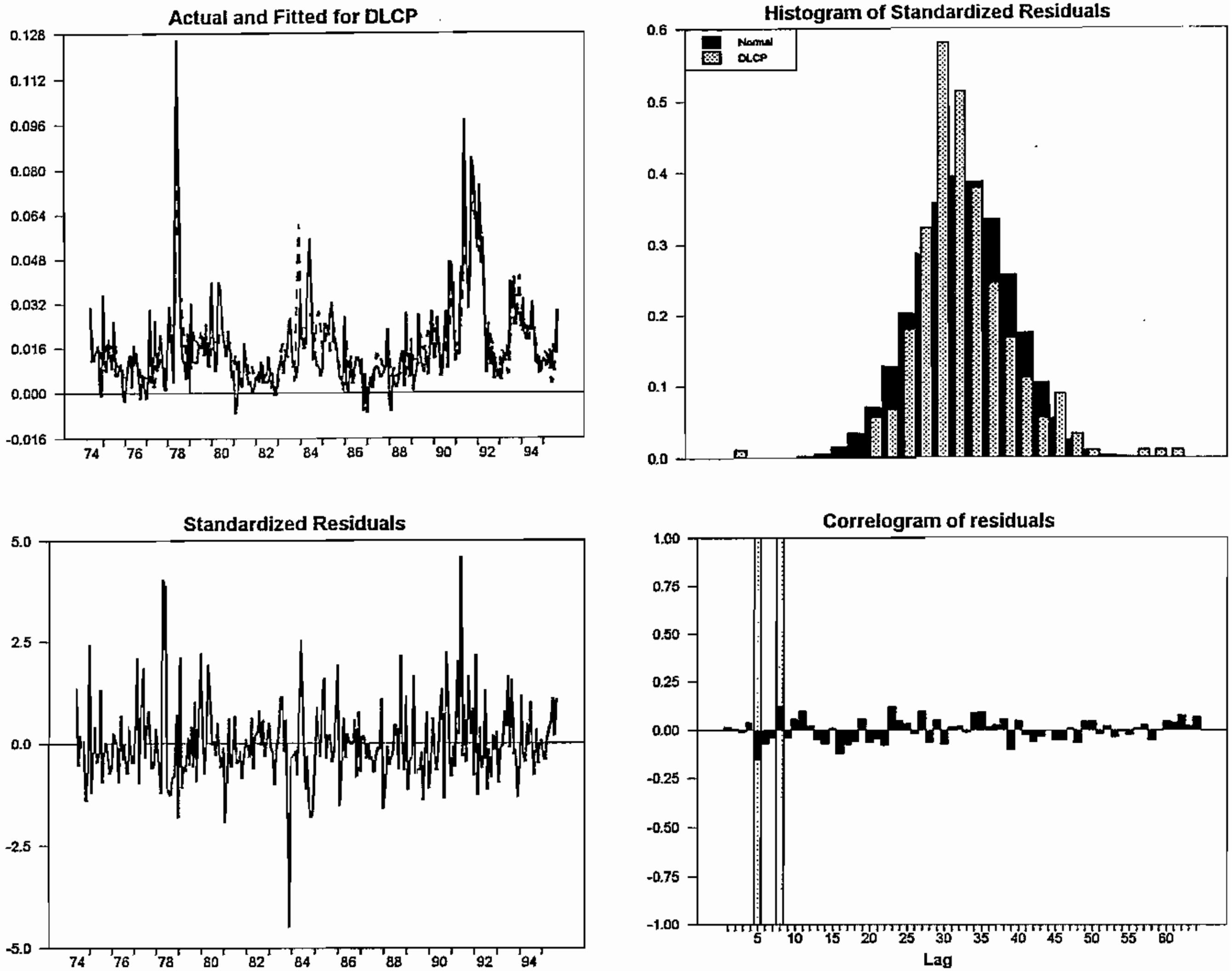

Correlogram of residuals
1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
-1.00
5 10 15 20 25 30 35 40 45 50 55 60
Lag

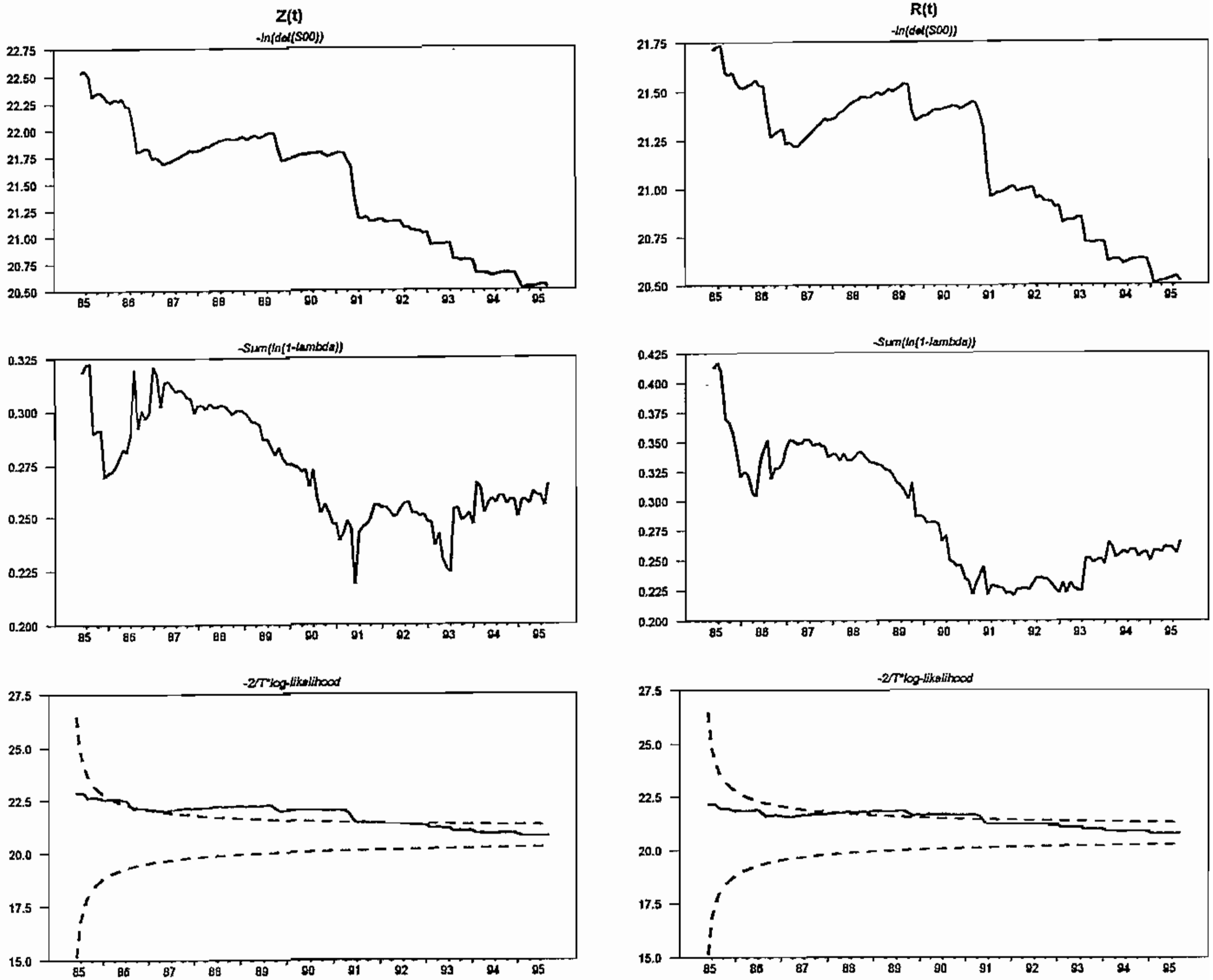

Actual and Fitted for DLVOL
4
3
2
1
0
-1
-2
-3
-4
74 76 78 80 82 84 86 88 90 92 94


Histogram of Standardized Residuals
Normal
DLVOL
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00


Standardized Residuals
4.8
3.6
2.4
1.2
0.0
-1.2
-2.4
-3.6
74 76 78 80 82 84 86 88 90 92 94


Correlogram of residuals
1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
-1.00
5 10 15 20 25 30 35 40 45 50 55 60
Lag


Actual and Fitted for DLCP
0.128
0.112
0.096
0.080
0.064
0.048
0.032
0.016
0.000
-0.016
74 76 78 80 82 84 86 88 90 92 94
Histogram of Standardized Residuals
Normal
DLCP
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Standardized Residuals
5.0
2.5
0.0
-2.5
-5.0
74 76 78 80 82 84 86 88 90 92 94
Correlogram of residuals
1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
-1.00
5 10 15 20 25 30 35 40 45 50 55 60
Lag



Z(t)
R(t)
-ln(det(S00))
-Sum(ln(1-lambda))
-2/T*log-likelihood
85
86
87
88
89
90
91
92
93
94
95