Sustainable Fiscal Strategies under Changing Demographics*

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Abstract: This paper develops an overlapping generations model to evaluate, first, the steady state growth-maximizing level of public debt around which the economy needs to stabilise; second, how the optimal level of public debt varies as a function of key population parameters; third, how fiscal rules designed to stabilise the economy around that debt level need to vary with the population parameters; and, fourth, how the model performs as a reasonable and plausible representation of the economies that we might be concerned with. Finally, following the diminished fiscal space and flexibility that is created by deteriorating population parameters, some political economy perspectives are offered.

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1. Introduction

General fiscal rules are legislative agreements intended to mitigate the deficit bias usually associated with fiscal policy and typically due to myopia by governments. Recent empirical research suggests that national fiscal rules are helpful in achieving greater budgetary discipline (Debrun et al., 2008; Nerlich and Reuter, 2013; Foremny, 2014). The issue about which specific rule is most effective in promoting fiscal discipline has recently attracted scholarly attention. For example, Bergman et al. (2016) find that a combination of an expenditure rule and a balanced budget rule, or a combination of an expenditure rule, balanced budget rule and debt rule, give significant positive effects on the primary balance for virtually all levels of government efficiency.

In this paper, we take a different route. Rather than formulating generic rules designed to reduce the deficit bias, we set up specific rules aimed at maximising economic growth.[1] This enables us to condition those rules on population parameters and age-related spending to show the impact of population change on fiscal balances and debt. We ask: first, whether, and in what circumstances, changing demographics will affect the net fiscal position; and, second, whether it is acceptable to allow larger debt burdens, or whether tax or spending austerity is always needed when demographic change leads to pressure on public finances.

Then, rather than evaluating alternative forms of fiscal rules, we restrict attention to a rule for public debt. This is in line with previous work which argued that debt targets are generally superior to deficit targets for both theoretical and practical reasons (Hughes Hallett and Jensen, 2012).[2] But choosing a debt target is not a trivial task. A key issue is how to account for (the discounted value of) future spending liabilities. If the implicit liabilities created by ageing populations are ignored, the debt criterion will ignore the future revenues required to avoid default despite the obvious need to cover the benefits promised to existing and future beneficiaries.

This is the case for extending debt targeting rules to account for predictable demographic changes. Put differently, forward looking fiscal rules are needed to allow for future liabilities created by adverse demographics. The implication is that governments facing demographic change or the need for higher social spending, will have to adjust (most likely restrict) their

[1] Existing models in the literature with the aim of determining optimal levels of debt include Aschauer (2000), Aiyagari and McGrattan (1998) and Checherita et al. (2013).

[2] If debt targeting is preferred, the question is: what debt or debt-to-GDP ratio should be targeted (Auerbach, 2009)? Deriving optimal levels of public debt may involve several complicated trade-offs. For example, how should intergenerational equity be balanced against economic performance and long term fiscal sustainability?

fiscal plans to accommodate those changes. Hence, a key theme here is to make debt control forward looking by designing a rule where fiscal policy reacts, not only to changes in existing levels of debt, but also to anticipated changes in future liabilities.

This study provides a comparative static analysis. Within that framework, the paper makes several contributions.[3] First, we offer a formal evaluation of the optimal debt level around which the economy needs to stabilise.[4] Second, we study how the optimal level of debt varies with key population parameters. Third, we show how fiscal rules designed to stabilise the economy around that debt level need to vary with the population age, life expectancy, birth rate and rising welfare expenditures.

While general fiscal rules primarily address the deficit bias, in this paper we also look at specific policies to deal with ageing and lack of growth in the work force. Such policies imply at least three roles for public policy: i) to improve the incentives to raise children, to maintain the number of taxpayers and the ability to sustain a certain level of public debt; ii) to improve the volume and effectiveness of public capital, to boost productivity and growth; iii) to smooth the distributional implications of demographic shocks, to dampen concerns about increased income inequality within and across generations and to mitigate the negative effects of rising inequalities on growth. In this paper, we focus on all three factors because they explain why we can reach a benign steady state and fiscal position despite adverse demographics, an explanation missing in the literature so far, but is perhaps our key contribution here.

How and whether the government should incentivize child-rearing is a highly political question. Since the children of today are the workers of tomorrow, we consider the relationship between demographics and sustainable government finances. We analyze this topic by allowing social spending aimed at alleviating the private cost of child-rearing. We include the fiscal policy since growing populations have a feedback that can help maintain a certain level of public debt. More specifically, we model government expenditures related to child-rearing. We then analyze how these expenditures influence the economy.

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[3] In a companion piece, Hughes Hallett et al. (2017), we focus on the dynamics of how different economies might reach their new steady state. Here we focus on establishing that a feasible (from a political economy perspective) steady state exists before analysing how to reach it. It makes little sense to study the transition before we can show that an acceptable/sustainable steady state exists.

[4] The optimal debt level depends on the marginal productivity of public capital, defined as public spending on investment projects which are (a) productive, (b) have an identifiable rate of return and (c) have a longer horizon than consumption expenditures. Conceptually, this is clear-cut. But in practice there are often problems in separating public investment from public consumption in the data (Checherita et al., 2013).

We also consider public and private capital as a labour productivity-enhancing factor in production. Public capital can be interpreted as education, R&D and the social infrastructure that underpins human capital formation and leads to a more skilled workforce. Private capital, however, can also provide incentives for innovation and competition. The ratio of public to private capital is therefore key to economic prosperity. The government seeks to improve the volume and effectiveness of public infrastructure, helping to raise productivity and the economy's growth rate. Having a government with this mandate enables this study to present an optimal public debt policy which allows for demographic factors.

Lastly, we study the income distribution implications: how they relate to economic growth and debt levels. The underlying distribution issues are, first, capital and labour income shares; and then, the quality, and way in which public capital is deployed as a productivity enhancing device. The political economy trade-offs are then a matter of tracing how policies under demographic change impact growth rates and debt levels under different distributions of income and wealth.

The roadmap of the paper is as follows. In Section 2 we set out our analytical framework: an OLG model extended to allow for public debt. Section 3 derives the optimal debt-to-GDP ratio; section 4 outlines policies to manage demographic change, and in so doing introduces the political economy forces that underlie the main issue: intergenerational equity and transfers. Sections 5 and 6 provide a simulation treatment of the effects of population change on optimal debt. Section 7 then illustrates the policy and political economy trade-offs that underlie problems of this type. Section 8 concludes and offers suggestions for future research.

2. The model

We set out a model featuring overlapping generations, changing demographics, welfare spending, and productive public and private capital accumulation (Yakita, 2008; Bokan et al., 2016). The economy contains homogenous individuals and firms operating in perfectly competitive markets. Individuals have two periods in their lives, first as a worker and then as a retiree. To introduce population ageing and entitlement spending, people face a risk of dying in the transition from worker to retirement, so that not every worker will live a full two-period life. In each time interval, working people form the young cohort and surviving retirees the old cohort. Public investment decisions are made by a government that issues debt and levies taxes in order to fulfil its objectives.

2.1 Firms

The economy is composed of a large number of identical firms which produce a homo-genous product by utilizing the services of private capital and labour. The production technology for each firm j is characterized by constant returns to scale and labour augmenting productivity:

$$Y_{j,t} = AK_{j,t}^{\alpha}\left(h_t L_{j,t}\right)^{1-\alpha};\ 0 < \alpha < 1 \tag{1}$$

where $Y_{j,t}$, $K_{j,t}$ and $L_{j,t}$ denote, respectively, the output level, the private capital stock, and labour input in firm j for period t. Total factor productivity (TFP) is captured by a standard scale factor, A (which may be time dependent), labour augmenting productivity (not firm specific) is described by h_t, and α is the output elasticity with respect to private capital.

Given the assumption of perfectly competitive markets for production goods and inputs, the equilibrium rental rate of capital, r_t, and wage rate, w_t, can be found in the usual way as the solutions to the firm's profit maximization problem:

$$r_t = A\alpha\left(\frac{K_{j,t}}{L_{j,t}}\right)^{\alpha-1}(h_t)^{1-\alpha} \tag{2}$$

$$w_t = A(1-\alpha)\left(\frac{K_{j,t}}{L_{j,t}}\right)^{\alpha}(h_t)^{1-\alpha} \tag{3}$$

Equations (2) and (3) reflect a perfectly competitive environment, where the marginal product of an input equals its price.

The labour augmenting productivity factor, h_t, taken as given by the private sector, is specified along the lines of Kalaitzidakis and Kalyvitis (2004):

$$h_t = \frac{K_t^{\beta} G_t^{1-\beta}}{L_t};\ 0 < \beta < 1 \tag{4}$$

where G_t is the public capital stock and β is the productivity elasticity of private capital. The aggregate private capital stock and the aggregate labour input are, respectively, defined as $K_t = \sum_j K_{j,t}$ and $L_t = \sum_j L_{j,t}$. Expression (4) reveals that both private and public capital have productivity augmenting effects, with β establishing the strength of the augmenting effect of private versus public capital.

Moreover, given the symmetry of firms, the equilibrium pricing expressions (2) and (3) imply that all firms share the same optimal solution, such that the ratio of input choices of each firm is the same in equilibrium: $K_{j,t}/L_{j,t} = K_t/L_t$. This allows us to rewrite expression (1) in terms of aggregate output at time t:

$$Y_t = AK_t^{\alpha}(h_t L_t)^{1-\alpha} = AK_t^{\alpha+\beta(1-\alpha)} G_t^{(1-\beta)(1-\alpha)} = AK_t^{\omega} G_t^{1-\omega} \quad (5)$$

where $Y_t = \sum_j Y_{j,t}$ and $\omega \equiv \alpha + \beta(1-\alpha)$. The firm specific optimality conditions can then be rewritten in aggregate terms as:

$$r_t = A\alpha \left(\frac{K_t}{G_t}\right)^{\omega-1} \quad (6)$$

$$w_t = A(1-\alpha)\left(\frac{K_t}{G_t}\right)^{\omega}\left(\frac{G_t}{L_t}\right) \quad (7)$$

Finally, for the purpose of simplification, this model assumes that neither private nor public capital depreciates over time. We define the ratio of public-to-private capital, G_t/K_t, as X_t.

2.2 Households

The economy is populated by homogeneous individuals. The lives of individuals are divided into working and retirement periods. The working period is of fixed length. Individuals in the working period form the young cohort; those in retirement the old cohort. At the end of the working period, a fraction of the young agents die while the rest move into retirement. The probability of dying by the end of the working period is given by the hazard rate λ; the same for all agents. So the probability of being alive at the beginning of the retirement period is $1-\lambda$. As there is no third period, the size of the retired cohort is $(1-\lambda)$ times the size of the working cohort in the previous period. With N_t being the population of working-age people, the total population of the economy at t is equal to $N_t + (1-\lambda)N_{t-1}$.

Young individuals work, consume, save and raise children. The retirees do not have children but they consume based on the return on savings from the previous period. The representative individual's decisions in both periods are based on the maximization of life-time utility which is given by the following function in period t:

$$\ln c_t + (1-\lambda)\rho \ln d_{t+1} + \varepsilon \ln n_t \quad (8)$$

where c_t (d_{t+1}) is consumption in the working (retirement) period, n_t is the number of children the individual has, ε is the priority/weight of having children in a person's life-time utility, and ρ denotes the usual time discount factor, being a value between 0 and 1.

The time endowment for a young individual is normalized to 1. The model does not include leisure explicitly, but rather includes the child rearing time during which the young individual receives no wages.[5] Labour income is then allocated between current consumption, savings and tax payments. However, the individual receives a subsidy for child rearing which is also subject to taxation. This subsidy, denoted by ρ_w, is a fixed ratio of the wage rate, w_t. The tax rate is the same for wages and the child-rearing subsidy[6].

The budget constraint of the representative young agent at time t therefore reads:

$$(1-\theta_t)[w_t(1-zn_t)+\rho_w w_t z n_t] = c_t + s_t \tag{9}$$

where $z>0$ is the rearing time per child, and θ_t denotes the flat tax rate. Savings are denoted by s_t and are exclusively invested in purchasing annuity assets. In what follows we assume that not all of the individual's time endowment can go towards child rearing: $(1-zn_t)>0$.[7]

As in Blanchard (1985), we assume the existence of an actuarially fair insurance company operating in a perfectly competitive market for insurance. This insurance company collects savings from agents and invests them in private capital and/or government bonds. We assume that government bond purchases can crowd out private capital. The insurance company pays a return on the savings to the agents that survive into retirement. The retirees therefore receive a rate of return of $\frac{r_{t+1}}{(1-\lambda)}$ on their savings. The returns (not the principal) on savings are taxed at a rate equal to that on labour income. Thus, the second period budget constraint reads as follows:

$$\frac{1+(1-\theta_{t+1})r_{t+1}}{(1-\lambda)}s_t = d_{t+1} \tag{10}$$

The maximization problem of young individuals consists of choosing the optimal amount of savings and number of children to maximize life-time utility subject to the budget constraints for the working and retirement periods. The amount of savings directly influences the amount consumed in both periods. Meanwhile labour income not saved in the working period goes to

[5] Hence the desire to have children crowds out the time spent working. A more general approach might allow saving more to leave bequests or invest in education. But to do that, workers have to have the children and pay carers to rear the children while they are working extra hours to make the extra savings. The result is little or no ***net*** effect on incomes, savings or the fiscal position. So we treat these cases in a simpler way in Section 3 below.

[6] We can think of the child-rearing subsidy as costs that are covered by the government. As such, the subsidy accounts for the amount of income that would be spent by the individual on child-rearing. The subsidy is indeed set-up as a labour income augmenting subsidy. Because it is income that ends up not being spent on child-rearing, then this subsidy should be taxed, and taxed at the same rate as labour income. This interpretation forces us to to see the subsidy as something that diminishes out-of-pocket costs with child-rearing.

[7] $(1-zn_t)>0$ is guaranteed if the interval between the lower bound on ε in (15), and the upper bound on ε when $(1-zn_t)=0$, is nonempty. That interval is always nonempty if $z<1$, a restriction that has to hold since not all agents can spend all of their allotted life time rearing children and still survive. The parameter restrictions in this model therefore guarantee $(1-zn_t)>0$.

current consumption, c_t; while that saved goes towards consumption in retirement, d_{t+1}. The overall maximization problem now is:

$$\max_{c_t, d_{t+1}, n_t} \ln c_t + (1-\lambda)\rho \ln d_{t+1} + \varepsilon \ln n_t$$

$$s.t.\ (i)\ c_t + s_t = (1-\theta_t)[w_t(1-zn_t) + \rho_w w_t z n_t]$$

$$(ii)\ d_{t+1} = \frac{1+(1-\theta_{t+1})r_{t+1}}{(1-\lambda)} s_t$$

By rearranging the first order conditions for this maximization, the following relationships are obtained:

$$\frac{d_{t+1}}{c_t} = \rho[1+(1-\theta_{t+1})r_{t+1}] \tag{11}$$

$$\frac{n_t}{c_t} = \frac{\varepsilon}{(1-\theta_t)(1-\rho_w)w_t z} \tag{12}$$

From these two conditions we find optimal solutions for savings, s_t, and number of children per individual, n_t:

$$s_t = \frac{(1-\lambda)\rho}{1+(1-\lambda)\rho+\varepsilon}(1-\theta_t)w_t \tag{13}$$

$$n_t = \frac{\varepsilon}{z(1-\rho_w)[1+(1-\lambda)\rho+\varepsilon]} \equiv n > 0 \tag{14}$$

Two insights follow from this. First, we see the fraction in equation (13) is solely composed of demographic related factors, meaning that savings are a fixed share of after-tax income *only* if population parameters do not change. Second, the same is true for the fraction in equation (14), except that it includes ρ_w which is determined by the government. Moreover, n is increasing in ρ_w. This should not be a surprise since increasing ρ_w lowers the financial burden of raising children, which gives individuals an incentive to have more children. The economy will continue to survive so long as $n_t \geq 1$, which holds true as long as:

$$\varepsilon \geq \frac{z(1-\rho_w)[1+(1-\lambda)\rho]}{1-z(1-\rho_w)} \tag{15}$$

This model is not designed to deal with cases of shrinking populations since it could not then produce a non-trivial steady state. For the sustainability of the steady state we assume that the condition in equation (15) holds. In fact, to ensure such a steady state exists, we do not restrict n to be greater than any particular replacement rate of the population, but impose a restraint on the preferences of agents for children such that it does. The reality is that, with immigration, population growth is positive (if small in some places) almost everywhere – as data from the *World Development Indicators* show. Revealed preference therefore justifies (15) as the appropriate restriction. Population growth may also be improved by changes in mortality (λ) as well as by fertility decisions (n, ε).

2.3 Government

The government collects taxes from income of the working population (through wages), the child-rearing subsidy, and from the returns on savings of the retired population. The tax rate is denoted by θ_t and is fixed regardless of income type. The government also issues public debt, b_t, and invests the proceeds in public capital, G_t. In addition, the government pays the child-rearing subsidy as specified above. The government budget constraint is therefore:

$$b_{t+1} = (1 + r_t)b_t + (G_{t+1} - G_t) + \rho_w w_t z n_t N_t \\ - \theta_t(w_t L_t + \rho_w w_t z n_t N_t + r_t s_{t-1} N_{t-1}) \tag{16}$$

We assume interest payments and public consumption are financed via taxes on wages, subsidies and returns on savings, as captured by the period-by-period budget constraint:

$$r_t b_t + \rho_w w_t z n_t N_t = \theta_t(w_t L_t + \rho_w w_t z n_t N_t + r_t s_{t-1} N_{t-1}) \tag{17}$$

Public debt is then issued to finance public capital formation, that is:

$$b_t = G_t \tag{18}$$

The model is thus stated in terms of the "golden rule" for public finance, so the government only borrows to invest, not to finance its consumption or transfer payments.[8] The question becomes: what is the optimal level of public debt and how is it determined?

3. The optimal level of public debt

3.1 Growth maximising public debt

The representative insurance company, and hence the young generation, can invest in both private and public capital. Equilibrium in capital markets requires that

$$s_t N_t = K_{t+1} + G_{t+1} \tag{19}$$

We can use conditions (17), (18) and (19), and combine them with capital rents and wages, (6) and (7), as well as the fact that $L_t = (1 - zn)N_t$ (labour supply equals the time endowment less child rearing time), to derive an expression for the income tax rate, θ_t, needed to satisfy current public spending:

[8] The rationale for this golden rule is given in Blanchard and Givazzi (2002) and Fatas et al. (2003). It is practiced in Germany, the UK and several other economies.

$$\theta_t = 1 - \frac{1}{\alpha X + \rho_w(1-\alpha)zn/(1-zn)+1} \tag{20}$$

Using the equilibrium condition for capital markets (19) and the solution for s_t in (13), we obtain:

$$\frac{(1-\lambda)\rho}{1+(1-\lambda)\rho+\varepsilon}(1-\theta_t)w_t N_t = K_{t+1} + G_{t+1} \tag{21}$$

This equation determines the dynamic relationship between wages, working population, and public and private capital. We define the balanced growth path as a situation in which public and private capital grow at a constant rate. Specifically, let the aggregate growth rate γ^A be defined as:

$$\frac{G_{t+1}}{G_t} = \frac{K_{t+1}}{K_t} = \frac{Y_{t+1}}{Y_t} = \gamma n \equiv \gamma^A \tag{22}$$

where n is the constant growth rate of the population and γ is the aggregate growth rate *per capita*. This now implies that the public-to-private capital ratio is constant in steady state. It also implies that the tax rate, θ_t, and the interest rate, r_t, are constant in steady state.

Next we use equation (21), together with (7), (17) and (20), to obtain a relationship from which we can derive a closed form expression for the economy's growth rate (see Appendix A for the full solution and definition of $\tilde{C}$):

$$\gamma^A = \frac{A\tilde{C}[(1-\alpha)/(1-zn)]}{X^{\omega-1}(X+1)(\alpha X + C + 1)} \tag{23}$$

In order to derive the public-to-private capital ratio which maximizes the aggregate growth rate along the balanced growth path, we take the first derivative of equation (23) with respect to the public-to-private capital ratio, X, set it equal to zero, and solve for X. The result is a general solution of the form (for a full derivation, see Appendix A):

$$X_{1,2} = \frac{-\omega\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1+\alpha\right) \pm \sqrt{\omega^2\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1+\alpha\right)^2 - 4\alpha(1+\omega)(\omega-1)\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1\right)}}{2\alpha(1+\omega)} \tag{24}$$

With our parameter restrictions, it is easy to show that the positive solution to this equation is also positive (see Appendix B):[9]

$$X^* = \frac{-\omega\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1+\alpha\right) + \sqrt{\omega^2\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1+\alpha\right)^2 - 4\alpha(1+\omega)(\omega-1)\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1\right)}}{2\alpha(1+\omega)} \tag{25}$$

Given equation (5), the optimal debt to GDP ratio in this model is now:

[9] We are only interested in a positive solution since capital stock ratios must be nonnegative.

$$d^* = \frac{1}{A} X^{*\omega} \tag{26}$$

3.2 Optimal debt policy and demographic factors

It is now apparent that the optimal debt policy dependens on the child-rearing subsidy rate. This subsidy rate is one major component in the balanced-budget tax rate. In this way, the subsidy plays a central role in the savings-investment nexus. It is also affecting fertility choices and hence the population growth rate. Therefore, the subsidy rate is a part of optimal debt policy. Furthermore, by maximizing aggregate growth per capita, $\gamma = \frac{\gamma^A}{n}$, identical results also hold for maximal aggregate growth, γ^A: equations (24) - (26). This implies the optimal level of the government debt does not affect childbearing decisions of agents. But optimal debt is influenced by childbearing through the subsidy, ρ_w. Hence demographics affect the optimal debt ratio; but the debt ratio does not affect population growth.

As we might expect, in the special case without subsides ($\rho_w \to 0$), the optimal debt ratio becomes independent of the population parameters; it becomes solely dependent on the elasticities of private capital to public capital (ω) and labour (α). It is important to note that the choice of ρ_w is not treated as a policy tool in this analysis, in the sense that the analysis does not attempt at deriving an optimal subsidy policy. Policy optimality would depend on efficiency/welfare concerns in relation to an endogenous population growth. Instead, the choice of ρ_w is a parameter used to explore different calibrations. Likewise, the government does not manipulate the child-rearing subsidy to promote fertility, though the analysis synthetically shows the economic effects of doing so. The ultimate focus is on providing the growth-maximizing optimal debt-to-GDP ratio for a *given* child-rearing subsidy (among other factors).

Moreover, the structure of the labour productivity-enhancing factor in production is key to the optimality of debt policy. Notice that if public capital becomes irrelevant relative to private capital in the labour-augmenting process h_t in equation (4), we have that $\beta \longrightarrow 1$, and consequently $(1-\omega) \longrightarrow 0$. In this case, both the optimal public-private capital ratio, X^* in equation (25), and the optimal level of debt, d^* in equation (26), tend to zero. This is in line with Greiner (2010) and Checherita et al. (2013).

It is worth noting that the subsidies described above can be interpreted in different ways to cover different types of age-related spending and social support. Suppose the subsidies take the form of support to raising children. For example, the UK provides child benefit payments which were taxed at standard rates until the 1980s and again from 2012. In fact, any grant which is either taxed or means tested can be written as equation (9) under uniform grant or means test rates. Most tax codes, including those in the US, offer a tax free element per-child equivalent to a net income supplement of $\rho_w(1-\theta_t)wzn$, which is then taxed at the standard rate (where ρ_w is set to make this expression equal to the tax saving).

Subsidies could also be in the form of support to higher education. The US, for example, taxes certain training grants and fee waivers. Subsidised loans or means tested tax reductions on fees operate like child benefits, except that z now refers to time spent in education (child rearing by society, rather than by parents). Subsidies could also support education in general, where state spending per pupil is related to the average wage and taxes (levied at standard rates on steady state earnings) that fund that spending. In this case, z is the proportion of the young population in state funded schools.

Moreover, subsidies could be regarded as health care costs, where care givers are paid through a state subsidy; or where, as in the US, those costs contain hidden subsidies. Another possibility is sick pay. For example, most EU countries pay a fraction of the wage for time off sick, but tax it as income. Similarly, care givers may be paid through a state subsidy, but taxed at standard rates.

4. Policies to manage demographic change

This section examines how two major demographic trends, reduced mortality and reduced fertility, affect fiscal policy and optimal levels of public debt. These two trends lead to ageing populations. In addition, we study the debt responses to an increase in preferences for current consumption over future consumption, as has been observed for the baby-boomer generation throughout the OECD area. The final analysis considers changes to public support of child-rearing.

4.1 Increased longevity

Within the framework developed above, we first evaluate how a lower probability of dying in the first period of life impacts the debt-to-GDP ratio. By differentiating the optimal debt-to-GDP ratio in equation (26) with respect to λ, we get:

$$d_\lambda^* = \frac{1}{A}\omega {X^*}^{\omega-1} X_n^* n_\lambda \tag{27}$$

where X_n^* denotes the first derivative of X with respect to n; and n_λ is the first derivative of n with respect to λ. Note that equation (14) implies $n_\lambda > 0$.

In order to determine the sign of the partial derivative X_n^*, we introduce a shorthand notation to write the solution for X^*. Let $f(n) = \rho_w \frac{(1-\alpha)zn}{1-zn} + 1 + \alpha$, so $f(n) - \alpha = \rho_w \frac{(1-\alpha)zn}{1-zn} + 1$. We define $c = -\frac{\omega}{2\alpha(1+\omega)}$. We now rewrite X^* in equation (25) as

$$X^* = cf(n) - \frac{c}{\omega}\sqrt{\omega^2 f^2(n) - 4\alpha(\omega^2 - 1)[f(n) - \alpha]} \tag{28}$$

Now let $\alpha F(n) = f(n)$, so that

$$X^* = c\alpha F(n) - \frac{c}{\omega} 2\alpha\omega \sqrt{\frac{1}{4}F^2(n) - [F(n) - 1] + \frac{1}{\omega^2}[F(n) - 1]}$$

After taking the first order derivative with respect to n, we get:

$$X_n^* = c\alpha F_n(n) - \alpha c \frac{1}{\sqrt{\frac{1}{4}F^2(n) - [F(n) - 1] + \frac{1}{\omega^2}[F(n) - 1]}}\left[\frac{1}{2}F(n)F_n(n) + \left(\frac{1}{\omega^2} - 1\right)F_n(n)\right]$$

Now let $G(n) = \frac{1}{2}F(n) + \left(\frac{1}{\omega^2} - 1\right)$. Rewrite the denominator above as,

$$\sqrt{\frac{1}{4}F^2(n) + [F(n) - 1]\left(\frac{1}{\omega^2} - 1\right)} = \sqrt{\left[\frac{1}{2}F(n) + \left(\frac{1}{\omega^2} - 1\right)\right]^2 - \left(\frac{1}{\omega^2} - 1\right)^2 - \left(\frac{1}{\omega^2} - 1\right)}$$

so that
$$X_n^* = c\alpha F_n(n)\left[1 - \frac{G(n)}{\sqrt{G^2(n) - \left(\frac{1}{\omega^2} - 1\right)^2 - \left(\frac{1}{\omega^2} - 1\right)}}\right]. \tag{29}$$

To see the sign of X_n^*, we need to determine the signs of c, F_n and the expression in brackets. Since $c = -\frac{\omega}{2\alpha(1+\omega)} < 0$, and $F_n(n) = \rho_w \frac{(1-\alpha)z}{\alpha(1-zn)^2} \geq 0$, the expression in square brackets is negative because the numerator is larger than denominator. Thus:

$$d_\lambda^* = \underbrace{\frac{1}{A}\omega X^{*\omega-1}}_{>0}\underbrace{X_n^*}_{>0}\underbrace{n_\lambda}_{>0} \geq 0 \tag{30}$$

This implies that rising life expectancy, in the form of a lower probability of death in the first period, leads to a fall in the optimal debt ratio for the economy.[10] The reason is straightforward. From equation (13), a fall in λ leads to a rise in savings by the working population ($\partial s_t/\partial\lambda < 0$) that realizes it will now have to save more for their retirement than before, at the

[10] A variation on this result is that a rise in the retirement age would be reflected in a rise in λ, as fewer agents would survive into the retirement period. Unlike the previous example, higher debt is now required to finance the extra output agents would need to produce during their working lives.

cost of less current consumption and fewer children. With lower population growth and higher longevity, it is no longer optimal for the government to sustain the same level of public debt as before. On one hand, it is costly for the government to raise public debt, due to increased interest payments and increased equilibrium interest rates. On the other hand, higher public debt, and thus, public investment implies higher wage rates. As such, when total private savings naturally increase due to increased longevity and *ceteris paribus* increased wage rates (lower fertility today implies less workers tomorrow), there is less of a need for the government to positively influence wage rates in the pursuit of maximal economic growth. Thus, the optimal public-private capital ratio becomes lower with lower fertility and higher longevity.

Notice also that when the debt-to-GDP ratio decreases, interest rates decrease. This worsens the inter-temporal substitutability from present consumption to future consumption. Thus, following a growth maximizing path for public debt provides an automatic stabilizer because government policy mitigates the individual choice of transferring a higher but sub-optimal growth in consumption into the future.

To establish the intuition behind the child subsidy effect on individual choices, notice from eqn. (7) that the aggregate wage bill (wL) is a function of X and public capital G. Hence, for constant X and G, any decline in L would increase the real wage (w) proportionally such that the wage bill is constant. This is of course a general equilibrium effect that does not affect labour supply incentives.[11]

Ceteris paribus, if a parameter change reduces L and increases the birth rate permanently, pre-tax incomes in the young generation do not decline. Instead, the tax rate will increase as the subsidy increases. As a consequence, individuals obtain more utility from non-production sources (children) at the expense of a lower disposable income for consumption or saving. On the other hand, this effect is zero if $\rho_w = 0$. Thus, we see that a positive subsidy makes an important difference to the consumption-saving capabilities of the young generation.

4.2 A fall in the birth rate

We next consider the case of a fall in the number of children per working family, equivalent to a fall in the birth rate captured by a fall in ε. In this case we can write, upon differentiating the optimal debt-to-GDP ratio (26) with respect to ε,

[11] If the constant population growth rate affecting L and G is positive, a one-time decline in L in the balanced growth equilibrium would still generate a permanent increase in w.

$$d_\varepsilon^* = \frac{1}{A}\omega X^{*\omega-1} X_n^* n_\varepsilon \tag{31}$$

where d_ε^* denotes the derivative of d with respect to ε; and where

$$n_\varepsilon = \frac{1+(1-\lambda)\rho}{z(1-\rho_w)[1+(1-\lambda)\rho+\varepsilon]^2} > 0 \tag{32}$$

which confirms that a fall in the preference for children leads to a fall in the birth rate. We may therefore work with change in either ε or n. It follows that

$$d_\varepsilon^* = \underbrace{\frac{1}{A}\omega X^{*\omega-1}}_{>0}\underbrace{X_n^*}_{>0}\underbrace{n_\varepsilon}_{>0} > 0 \tag{33}$$

So, for positive subsidies, a fall in the birth rate should lead to a lower optimal debt level. A lower preference for children permits agents to increase their labour force participation due to less time spent rearing children. That leads to an increased taste for consumption and savings. Additionally, the drop in fertility eases the burden of the child-rearing subsidy leading to drops in the tax rate, the optimal debt level, and ultimately to a rise in aggregate GDP growth per capita.

As in the case of greater longevity, this makes intuitive sense because it means a smaller number of people will be available to service the debt repayments in the future. Thus, at the same time as the individuals' desire for saving increases, the government's lower debt to GDP level marginally worsens the conditions for transferring of current consumption into the future due to the interest rate effect.

4.3 Higher discount factors

The third example is to consider the effect of an increase in the discount factor, taking the form of a fall in ρ, and often referred to as the baby boomer problem. Indeed, this is exactly what has happened with the baby-boomer generation in Europe, albeit less dramatically than in the US or UK where the preference for current consumption over future consumption increased and the savings rate fell.[12] Those are the changes implied by (8), which in turn implies $\partial s_t/\partial\rho > 0$ from (13).

To examine the debt consequences of this case, we need to determine the sign of

$$d_\rho^* = \frac{1}{A}\omega X^{*\omega-1} X_n^* n_\rho \tag{34}$$

where, by eqn. (14),

[12] This example assumes social support (ρ_w) is unchanged. If not, savings may increase – as in Japan or China.

$$n_\rho = \frac{-\varepsilon(1-\lambda)}{z(1-\rho_w)[1+(1-\lambda)\rho+\varepsilon]^2} < 0. \tag{35}$$

Hence we have:

$$d_\rho^* = \underbrace{\omega X^{*\omega-1}}_{>0}\underbrace{X_n^*}_{>0}\underbrace{n_\rho}_{<0} < 0 \tag{36}$$

In words, a lower concern for the future (the revealed preference of the baby boomer generation) should lead to higher levels of public debt. That is exactly what has happened in nearly all European and OECD countries over the past two decades, although baby-boomers may not have been the only reason.

The intuition is that baby boomers have lower preferences for saving and higher preferences for immediate gratification, which in this model is present consumption and child rearing. Consequently, the birth rate increases and so does the government's debt capacity. As such, even though individuals save less now, the public capital stock rises relative to that of private capital (a higher public debt to GDP ratio). At the same time subsidy receipts increase the income available at a younger age and the increase in public debt marginally increases the interest rate, thus improving the transfer of consumption from the present to the future.

This poses a policy challenge: not only do current levels of debt need to reduce to normal levels, but they should be allowed to fall further as the baby boomers retire. This might be taken as justification for imposing a "granny tax" on retirees in cases where social support and excess debt are allowed to continue unchecked. However, a proper analysis of that case would need a model with separate tax rates for young and old, since those rates may affect working and saving behaviour.

4.4 Increase in support for child-rearing

Finally, we consider the effects of ρ_w on the growth-maximizing debt. As an example, we highlight the effects of a government policy aimed at increasing fertility by reducing the private cost of child-rearing. This could be the case when societies are faced with ferility rates below the replacement rate leading to declining populations.

We now need to determine the sign of $d_{\rho_w}^*$. Note that, through the chain rule, the differentiation of d^* w.r.t. ρ_w can be written as:

$$d_{\rho_w}^* = \frac{1}{A}\omega X^{*\omega-1}\left(X_{\rho_w}^* + X_n^* n_{\rho_w}\right) \tag{37}$$

where $X^*_{\rho_w}$ and n_{ρ_w} denote first derivatives with respect to ρ_w.

We already know the sign of X^*_n. We have left to determine the sign of n_{ρ_w} and $X^*_{\rho_w}$. For the first of the two tasks, we have that:

$$n_{\rho_w} = \frac{\varepsilon}{z(1+(1-\lambda)\rho+\varepsilon)(1-\rho_w)^2} > 0 \tag{38}$$

For the second task, in order to determine the sign of $X^*_{\rho_w}$ we can perform an identical analysis to the one done for X^*_n in sub-section 4.1, except that now we define instead $f(\rho_w) = \rho_w \frac{(1-\alpha)zn}{1-zn} + 1 + \alpha$, $\alpha F(\rho_w) = f(\rho_w)$, and $G(\rho_w) = \frac{1}{2}F(\rho_w) + \left(\frac{1}{\omega^2} - 1\right)$. With these definitions, we can write:

$$X^*_{\rho_w} = c\alpha F_{\rho_w}(\rho_w)\left[1 - \frac{G(\rho_w)}{\sqrt{G^2(\rho_w) - \left(\frac{1}{\omega^2}-1\right)^2 - \left(\frac{1}{\omega^2}-1\right)}}\right] \tag{39}$$

In the same fashion as before, to see the sign of $X^*_{\rho_w}$ we need to determine the signs of c, F_{ρ_w} and the expression in brackets. We still have that $c = -\frac{\omega}{2\alpha(1+\omega)} < 0$ and that the expression in square brackets is negative because the numerator is larger than denominator. But now we have a different expression to evaluate: $F_{\rho_w}(\rho_w) = \frac{(1-\alpha)zn}{\alpha(1-zn)}$. The numerator of this expression is always positive. As for the denominator, since we have assumed that not all of the individual's time endowment goes towards child rearing, i.e. $(1 - zn_t) > 0$, we have that $F_{\rho_w}(\rho_w) > 0$. Thus, we reach the conclusion that $X^*_{\rho_w}$ is positive:

$$d^*_{\rho_w} = \underbrace{\frac{1}{A}\omega X^{*\omega-1}}_{>0}\left(\underbrace{X^*_{\rho_w}}_{0} + \underbrace{X^*_n n_{\rho_w}}_{>0}\right) \tag{40}$$

It follows from equation (40) that the optimal debt burden in equation (25), d^*, will increase with subsidies, ρ_w. The subsidy rate reduces the cost of raising children, so higher subsidies lead naturally to higher fertility, *n*, by equation (14). On the other side, a higher subsidy rate is covered by an increase in the tax rate, θ_t by equation (20), which reduces the available flow of funds from savings to investment described by equation (21). Even so, due to an increased fiscal capacity stemming from a permanently higher number of workers in the economy, it is optimal for the government to induce a higher level of public capital *relative* to private capital.

5. Calibration

To test the performance of our model as a reasonable and plausible representation of the type of economy that we might be interested in, we first calibrate it to match the typical advanced OECD economy. This allows us to assess fiscal sustainability under social or demographic changes of different sorts; how that compares to the fiscal position and growth rates in OECD economies in their current or pre-financial crisis state; and whether these economies would find it easy or feasible to transition to the corresponding steady state.[13]

5.1 Production parameters

To gain some insight into the current state of OECD economies facing demographic change, we perform a baseline calibration of the model as set out in section 2. The calibration is based on the period length of 45 years. This assumes that agents in the model start working at age 20, work for 45 years and retire at age 65. Child rearing can be shared by any working adult pair. In terms of income foregone, child rearing time per child is therefore z=0.2=9/45 (the age of majority is reached at 18) and corresponds to 9 years of each parent's working life.

We choose the remaining parameters with several key factors in mind. Primarily, we seek to emulate a realistic demographic structure for the typical OECD economy. Table 1 sets out the parameter values, while the motivation for each value follows below.

---Table 1 about here---

We focus first on production. The value of the output elasticity of private capital, α, should be around 0.25. This is in line with results presented elsewhere (e.g., Felipe and Adams, 2005). β captures the private sector share in productivity gains: that is, the share of any income gains that come from productivity increases due to private capital as opposed to public capital. In this calibration, the parameter β is set at 0.6. The TFP scale effect A is subject to interpretation. Several authors (e.g., Hviding and Merette (1998) and Fehr et al. (2013)) argue that it should reflect (TFP) productivity effects; and those operating with calibrated models proceed in two steps as we do here: first to establish a steady state solution, and then examine the features of the transition path to that steady state. In that case, A would be time varying: $A_s = A_0(1.01)^s$, where A_0 is the value of A at the start of generation t; TFP growth is set at 1% per year; and s={0, 1, 2 …..45} denotes the year within generation t. The initial parameter A_0 must then be normalised to align the model with historical data. In our case, this is the debt-to-GDP ratio at the start of the calibration period. For the OECD, this yields $A_0 = 48.5$ (see Table 5) and is calculated as follows. First, the model generates a debt ratio, eqn. (26), from the stock of public

[13] In later work, we can go on to look at the steady states implied in particular economies, including: (1) the "Anglo-Saxon economies" (US, Canada, UK); (2) countries where the process of population ageing is particularly strong (Japan, Italy, Germany); (3) Nordic welfare states with flexible markets and strong social provision (Denmark, Sweden); (4) economies with slow population growth and limited social support (Russia, China); and (5) those with young and fast growing populations (India, Mexico or the Philippines).

capital/debt and output accumulated over a single generation (45 years). We need to convert that to a steady state debt ratio for a single year (the stock of public debt outstanding at *s,* relative to output in year s). We estimate the latter as the average annual output in a generation, ($Y_s = Y_t/45$), and apply $A_0(1.01)^{22.5}/45$ to the denominator of eqn. (26) to get the final year estimate. This is a crude estimate, but it is all that can be done within the confines of an OLG model.

5.2 Preference and demographic parameters

Parameter ρ is designed to capture the value an agent puts on future consumption relative to present consumption; that is, the discount to be applied at the start of the working period to the decisions made in the retirement period 45 years later. A constant compounded annual discount rate of 0.975 implies ρ=0.32 for a planning period spanning four and a half decades.

The values of 0.2 and 0.15 for z and ρ_w, respectively, imply a *net* loss of close to 17% in life-time earnings per child born. According to the United States Department of Agriculture, the net expenditure per child in a middle income household (from birth until age 18) is estimated to be \$245340, or \$13630 annually for children born in 2013; while, according to the Current Population Survey of the US Census Bureau, the mean household total money income in the year in 2014 was \$72641.[14] This implies that the direct cost of each child is approximately 17% of lifetime income, suggesting a subsidy rate of $\rho_w = 0.15$.

Next ε and λ are the key demographic parameters; both influence the age structure of the population. Regarding ε: we set this equal to 0.3, so that the number of children born per adult comes out at n=1.25, by eqn. (14), implying the number of children per woman is 2.5. This implies a slow growing (*native*) population, growing at 0.5% per year (24.9% over 45 years). These figures then match the experience of the average EU economy rather closely, see Table 2.

---Table 2 about here---

Regarding λ: the number of people aged 65+ relative to the population of working age (20-64) in the average OECD economy is 28%, see table 3. In the context of our model, this implies that the probability of surviving into the retired cohort of 65+ is about 0.35. This yields an age-dependency ratio of 28%, as in table 2, for the average OECD economy.

[14] See Lino (2014) and DeNavas-Walt and Procter (2014) respectively.

---Table 3 about here---

Notice that $\varepsilon = 0.3$ ensures that a steady state exists; the lowest permissible value, by (15), is $\varepsilon \approx 0.23$. We make an important distinction between reality and the model here. The model does not recognise that not every woman is able or willing to have children. The rule of thumb is that 10% of adults do not have children which means that, to sustain a constant population, each woman needs to have 2.2 children. What we are doing in this paragraph is checking that our calibration not only produces a steady state in the model ($\varepsilon \geq 0.23$), but also a realistic steady state population growth of 0.49% per year ($\varepsilon = 0.3$).

6. Steady state characterisation

6.1 A first look at the results

Table 4 summarizes the output of the baseline simulation. We see that the demographic parameters, viz. the number of children per agent and the age dependency ratio, are in line with that expected in a typical OECD economy, as represented in Tables 2 and 3. The optimal debt to GDP ratio, d^*, is at 40.16% which is a reasonable value for the current state of typical OECD economies. The ratio of public-to-private capital is roughly 30%, while the growth rate of output is only 85.35% per working period (45 years). This *per period* growth implies an *annual* growth rate of 1.38% on average in steady state. However, this model does not include any growth in total factor productivity TFP as such. So the average growth in output per head of 0.88% or less is driven by accumulated savings, matching the zero long term per capita growth rates predicted by almost all standard growth models, thus confirming that we are at or close to steady state after one generation. Allowing exogenous annual growth of 1% in TFP in addition, we arrive at a conventional figure of 2.39% for overall growth per year.

---Table 4 about here---

What are the lessons from these simulation results? First: a steady state solution, and hence a sensible and sustainable set of fiscal policies for economies facing significant demographic change, is both feasible and available. Second: based on reasonable parameter values for the average OECD economy, the steady state solution demonstrates very reasonable properties given the post financial crisis experience in the OECD. It suggests growth around the 2%-3% mark as the “new normal”, rather than the 4%-5% observed in the 1990 to 2005 era.

It also suggests that a public debt level of 40% of GDP is a reasonable debt level to aim at.[15] This accords well with the Maastricht Treaty/Fiscal Compact upper limit of 60% to create a debt target with a safe zone to accommodate shocks. Moreover, it also accords well with the pre-crisis experience of non-Eurozone economies (the US or UK) where debt was held in the 35%-45% range. Finally, it suggests a mixed economy with a public sector component of 30%, lower than in Scandinavia or the Eurozone (France, Italy, Germany or the Netherlands), but similar to that in the US, Canada or the UK in the pre-crisis era. The growth in population is close to that projected for the OECD (Table 2, last column).[16]

A first impression of these results is that the demographic situation is perhaps not as bad as some commentators fear. But it is no easy task for most OECD countries to get themselves down to this optimal steady state level of debt (40%; or 70% off current debt ratios, see Table 5) and also to maintain this steady state rate of growth in productivity, which currently is a matter of considerable concern to the OECD. In addition, endogenous growth under this demographic burden is slow (≈1.4%); a larger share of the growth comes from TFP.

These results seem to reflect the current OECD experience pretty well, suggesting that the slow growth problem is chiefly due to low productivity rather than poor market performance. Interestingly, our results are consistent with the standard growth models showing that GDP growth ultimately comes from either growth in productivity or growth in the workforce. Since we have little of the latter (n=1.25 or 2.50 per woman, is only a little above the constant population replacement rate), GDP growth has to come from productivity growth. Ultimately, demographics should not affect that: the population growth is simply too slow to make up the difference. However, the transition to steady state growth could become more difficult for economies that currently find themselves a long way from that steady state.

6.2 Changes in the population parameters

To provide insight into the effect of demographic change on the steady state we evaluate the partial derivatives in Section 4, using baseline simulation results from Table 4. The results are presented in Table 6.

---Table 6 about here---

[15] With a range of 44%-60% in the Euro Area without TFP (30%-50% with TFP), or 36%-60% in the OECD.

[16] The calibration also yields an old-age dependency ratio of 28%, see Table 3 for 2015. This is very low and suggests a new a risk: if migrants arrive but then adopt the mores of the domestic population, as assumed by our model, then age-dependency ratios will rise without an increase in the long term population growth rate.

A general finding is that the changes in outcomes are all rather small with respect to d^*. Even doubling the largest of them, the importance of having children (ε), would add only 0.5 percentage points to the optimal debt ratio. This illustrates the scope of age-related transfers. Both ε and λ influence the optimal debt ratio through n and the child-rearing subsidy. By introducing more age related government expenditure the results are expected to be more pronounced.[17]

Thus, we conclude that not only is the optimal steady state under the new demographics relatively benign, but the likely future demographic and social *changes* will make little difference to that steady state once the economy gets there. In the long term, there is no great sensitivity to demographic or economic shocks. Instead, for most OECD economies, the real difficulty will be how best to manage the transition from where they are now to their optimal steady state.

6.3 Change in child-rearing subsidy

Changing the child-rearing subsidy rate also has an impact on the equilibrium steady state. As an example, we consider a case where the government uses the child-rearing subsidy to reverse an adverse demographic shock. Specifically, suppose a preference shift, such that agents have lower propensity to raise children, thereby directing their income towards consumption. In general, this could be problematic for society if the fertility rate drops below the replacement rate. When faced with this problem, the government might step in and raise the child-rearing subsidy. Indeed, we know from Section 4 that (a) a drop in the preference towards having children leads to a drop in the optimal debt-GDP ratio and (b) a higher subsidy rate leads to an increase in the optimal debt-GDP ratio.

To study this in further detail, we produce a calibration exercise where we look at the steady state effects of a composite shock where, first, there is a severe drop in the preference for children, leading to the fertility rate reaching the population replacement rate, and, second, a rise in the child-rearing subsidy as the government seeks to maintain a growing population, ultimately matching the fertility rate to its baseline value.

- *Table 7 around here -*

The results are summarized in Table 7. All parameters are the same as in the baseline calibration presented in Table 1, except for the explicit changes made to ε and ρ_w. For clarity,

[17] Some generalisations of the model may also be useful: removing the assumption of no capital depreciation, of equal taxes on wages and subsidies, or of subsidies as a fixed ratio to wages.

the tax rate has been unraveled into taxes associated with servicing the public debt and taxes levied for the coverage of subsidies. From the calibration results we see that a drop in preference for children decreases the total tax rate, since there is less strain on the government's budget, and increases savings as agents divert income away from child-rearing. The tax rate associated with debt servicing increases when the preference for children decreases. There are two opposing forces at play here. On one hand, optimal public debt decreases. On the other hand, the tax base decreases due to the fact that the subsidy payments are subject to taxation. Ultimately, the tax base effect is stronger resulting in an increase in the tax rate necessary for debt servicing. Economic growth *per capita* increases, even though economic output decreases because of fewer children.

When we account for the government's response of increasing the subsidy to reach the baseline value of fertility a different picture is drawn. The added government expenditure of the raised subsidy is covered by taxation, increasing the tax rate well above the baseline. This is solely due to the higher subsidy rate, since the tax rate associated with debt servicing actually decreases as a result of the broader tax base. The saving rate of after-tax income remains the same. The public-private capital ratio and thus optimal government debt is above the baseline. Interestingly, both per capita GDP growth and GDP growth are greater than in the baseline case. Per capita growth has, however, decreased compared to the case with an unchanged subsidy, which reveals a trade-off in fertility-promoting policies: Changes to the child-rearing subsidy have opposite effects on economic growth and economic growth *per capita*.[18]

In sum, the government's budget would be eased as the preference for children dropped but tightened, and tighter than before the preference shift, when implementing subsidy rates that raises fertility to its former level. This example demonstrates the trade-off between population growth and economic growth when determining the extent of the child-rearing subsidy. Finally, note that this analysis does not address the issues of an optimal value for the child-rearing subsidy. The analysis merely evaluates steady state policy effects.

7. Policy and the political economy trade-offs under demographic change

In order to evaluate the political economy implications of our fiscal imbalances, figure 1 provides isoquant lines for different values of growth and debt. The isoquants are determined by considering a different mix of values for α and β. In this sense income/wealth inequalities,

[18] More generally, one can show that, *ceteris paribus*, changes to the child-rearing subsidy always have opposing effects on economic growth and economic growth *pe capita*.

and who benefits from productivity gains, can be discussed in the wider context of economic performance and debt sustainability.

--- Figure 1 about here ---

The polical economy implications here are dependent on specific interpretations of α and β. As the value of β increases, the importance of private capital in labour augmenting productivity rises – as seen from equation (4). By contrast, a fall in β implies greater importance for public capital. At the same time, a rise in α increases the productivity of private capital and diminishes the share of labour in the production function, see expression (1). Hence higher values of α and β imply an economic system increasingly based on private capital, leading to a more unequal society. Lower values of α and β, by contrast, imply a society with stronger public infrastructure or higher social support, with publicly financed education and universal healthcare as classic examples.

Figure 1 can be interpreted as follows: If we go northeast up the diagram we get lower debt, but little additional growth, and none at all if the movement is mostly east, plus an increasing degree of income or wealth inequality. But the potential variations in growth are small. By contrast, moving northwest up the diagram brings higher growth more rapidly, with greater income and wealth equality and also lower debt ratios, although the debt reductions will become increasingly elusive (and may not materialize at all if there is too much movement west). In both cases, the income gains from higher productivity would increasingly accrue to the owners of capital, and necessarily so because to get any reductions in debt ratios we have to persuade the private sector to invest in productivity improving capital and production techniques, rather than public investment funded by public debt.

The implication of these steady state results is that public sector productivity is crucial and matters a great deal for economic performance. But more than that, it also matters how public capital is deployed. Second, there are a series of complicated and unwelcome trade-offs to be resolved. Greater income inequality clearly inhibits growth. But promoting productivity and growth through the public sector or infrastructure investment increases debt. Thus, an appropriately designed productivity policy can allow higher growth, or possibly lower debt, and with or without incurring greater inequality of incomes. But whichever route is chosen, that policy has to allow a greater share of the income gains made possible by higher productivity to go to the owners of private capital if higher debt ratios are to be avoided or the burden of public debt is to be reduced. These are the political economy trade-offs which arise when fiscal balances come under threat from deteriorating population dynamics.

Similarly, we can see that higher productivity can also be used to lower public debt, with or without increasing income inequalities. But it again comes at the cost of weaker/lower growth rates if income inequalities are allowed to increase too sharply. And a greater share of gains from this higher level of productivity will go to the owners of capital if we try to maintain or increase growth rates and employment at the same time. On the other hand, reversing this argument, policies designed to increase public participation in raising productivity will reduce growth but increase debt; whereas those designed to reduce income inequality directly usually increases both growth and debt.

Again the political economy trade-offs, in terms of economic performance and fiscal imbalances, are laid out in uncompromising reality once we take account of changing demographics and how the gains from productivity are distributed. It would appear that productivity plays a key role, perhaps more than we thought before. But the way productivity gains are used is at least as important as increased levels of productivity. And productivity only assumes this role because demo-graphic changes mean that we have to diminish their impact on fiscal imbalances without scaling back on entitlement spending – as shown in the model we have used.

8. Concluding remarks

The model and calibrated simulations in this paper show that the steady state outcomes for an advanced economy facing today's demographic and social trends are relatively benign and resemble those of the OECD economies a few decades back. Similarly, plausible changes to population parameters and social conditions in the future would make rather little difference to steady state outcomes. Instead the steady state is largely driven by economic fundamentals: productivity, the production structure, income generation, income inequality and distribution of productivity gains. The main problem is to get to the chosen steady state without undue strain or fiscal collapse on the way (Hughes Hallett et al., 2017).

Hence, our results suggest that demographic change is not necessarily a problem, if properly handled, compared to the kind of fiscal laxity we have seen over the past 20-30 years.[19] This is

[19] This result is however sensitive to the chosen calibration. For the average OECD economy, our results hold true. But an economy with other characteristics might face a more challenging situation than the average OECD economy. Specifically, our results are sensitive to the size of child-rearing subsidy, and, to some extent, to the

contrary to a conventional reading of the IMF's earlier advice. But it is still consistent with the IMF view if we reinterpret that to mean "this shows that ageing and social change is not a problem in itself, in steady state, so long as reliable fiscal rules or credible fiscal restraints are set up in advance to manage the rest of fiscal policy". In that sense, our work re-inforces the need for forward-looking fiscal rules that explicitly allow for the cost of demographic change.

But to imply that ageing and changing demographics are not a problem in steady state is not to say that all OECD economies will be able to get used to living in an era of slower growth and lower debt than they enjoyed for three or four decades past. In fact, the real problem is likely to be how to create and then safeguard the transition to that steady state. In short, the challenge is to find and maintain durable dynamic adjustment paths.

In future work, a focus on modeling the dynamics of transition and risks in adjusting from the current fiscally-weak-slow-growth of most OECD economies to the slow growth but fiscally sustainable steady state predicted by our model is warranted. Finally, income inequalities play a crucial role in determining the growth rates and debt reductions that are achievable under demographic and social change. This is a new aspect of the problem that deserves a more detailed investigation.

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values of α and ω. The higher the subsidy rate, the more impact demographics have on optimal steady state. The lower are α and β (greater income equality), the more sensitive is the steady state to demographic change.

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Appendix A: The solution to the optimal level of debt problem

We start from (22), with (21), (3) and (18), to obtain an expression from which we can derive a closed form expression for the economy's growth rate γ^A:

$$\widetilde{C}\left(\frac{1}{\alpha X+\rho_w(1-\alpha)\frac{zn}{1-zn}+1}\right)A(1-\alpha)\left(\frac{K_t}{G_t}\right)^{\omega}\frac{G_t}{L_t}N_t=K_{t+1}+G_{t+1} \tag{A1}$$

where $\widetilde{C}=\frac{(1-\lambda)\rho}{1+(1-\lambda)\rho+\varepsilon}$. This is equation (24) in the main text. Next we set $C=\rho_w(1-\alpha)\frac{zn}{1-zn}$ so that (A1) becomes

$$A\widetilde{C}\left(\frac{1}{\alpha X+C+1}\right)\frac{1-\alpha}{1-zn}\frac{1}{X^{\omega}}=\frac{K_{t+1}}{G_{t+1}}\frac{G_{t+1}}{G_t}+\frac{G_{t+1}}{G_t} \tag{A2}$$

Using (23) we can now rewrite (A2) as

$$A\widetilde{C}\left(\frac{1}{\alpha X+C+1}\right)\frac{1-\alpha}{1-zn}\frac{1}{X^{\omega}}=\gamma^A(1+\frac{1}{X})$$

This allows us to solve for the economy's aggregate growth rate, γ^A, to get

$$\gamma^A=\frac{A\widetilde{C}\frac{1-\alpha}{1-zn}}{X^{\omega-1}(X+1)(\alpha X+C+1)} \tag{A3}$$

which is the growth rate we need to optimize with respect to the public-private capital ratio.

To perform this optimisation, define $C_3=A\tilde{C}\frac{1-\alpha}{1-zn}$. Take the first order derivative of γ^A (denoted by $\hat{\gamma}$) with respect to X:

$$\hat{\gamma}(x)=-C_3\frac{(\omega-1)X^{\omega-2}\left[\alpha X^2+(\alpha+C+1)X+C+1\right]+X^{\omega-1}(2\alpha X+\alpha+C+1)}{\left[X^{\omega-1}(X+1)(\alpha X+C+1)\right]^2}$$

Setting this expression equal to zero, we can write

$$\alpha(\omega-1)X^{\omega}+(\omega-1)(\alpha+C+1)X^{\omega-1}+(\omega-1)(C+1)X^{\omega-2}+2\alpha X^{\omega}+(\alpha+C+1)X^{\omega-1}=0$$

which results in a quadratic equation for the optimal public-private capital ratio X:

$$\alpha(\omega+1)X^2+\omega(\alpha+C+1)X+(\omega-1)(C+1)=0$$

The required solution is therefore:

$$X_{1,2}=\frac{-\omega\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1+\alpha\right)\pm\sqrt{\omega^2\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1+\alpha\right)^2-4\alpha(1+\omega)(\omega-1)\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1\right)}}{2\alpha(1+\omega)}$$

which is (26) in the main text.

Appendix B: The positive root

We now demonstrate that our solution for X^* is in fact postive. We start from (27) in the text. Since $\omega>0, 0<z<1, n\geq 1, 0<\alpha<1$, it follows that $\omega\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1+\alpha\right)>0$ and $2\alpha(1+\omega)>0$.

Therefore to get a positive solution, $X^*>0$, we need the numerator of (27) to be positive. It will be positive if and only if

$$-\omega\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1+\alpha\right)+\sqrt{\omega^2\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1+\alpha\right)^2-4\alpha(1+\omega)(\omega-1)\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1\right)}>0$$

.

We therefore need to check if this condition is in fact satisfied for the parameter constraints that we have imposed. We require

$$\sqrt{\omega^2\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1+\alpha\right)^2-4\alpha(1+\omega)(\omega-1)\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1\right)}>\omega\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1+\alpha\right);$$

or

$$\omega^2\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1+\alpha\right)^2-4\alpha(1+\omega)(\omega-1)\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1\right)>\omega^2\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1+\alpha\right)^2;$$

or $$-4\alpha(1+\omega)(\omega-1)\left(\rho_w\frac{(1-\alpha)zn}{1-zn}+1\right)>0$$

which is true since $-4\alpha(1+\omega)(\omega-1)=-4\alpha(\omega^2-1)>0$ and $\rho_w\frac{(1-\alpha)zn}{1-zn}+1>0$ necessarily hold. Hence X^* in (27) is positive.

TABLES AND FIGURES

Table 1: Parameter values of the baseline simulation, OECD area

ρ	Discounting	0.32
ε	Importance of children in utility	0.30
α	Private capital's share in national income	0.25
β	Private share in gains from productivity	0.60
A_0	Initial Scale effects parameter	48.50
λ	Hazard rate	0.65
z	Rearing time per child	0.20
ρ_w	Child rearing subsidy rate	0.15

Table 2: Forecasts of population growth over 45 years.

	1970-2015	1995-2040	2015-2060
EU	15.59%	8.71%	2.90%
Denmark	15.34%	20.34%	15.70%
Germany	32.69%	-4.57%	-12.00%
Finland	18.58%	18.80%	13.90%
Sweden	21.78%	33.14%	34.28%
United Kingdom	16.79%	27.44%	23.68%
Norway	33.74%	63.43%	5.08%
Average of countries	**23.15%**	**26.43%**	**22.11%**

Source: Eurostat

Table 3: Forecasts of age dependency ratio (percentage).

	2015	2030	2050
Denmark	32	41	45
Finland	35	48	51
Germany	35	49	60
Norway	28	37	42
Sweden	34	41	45
United Kingdom	31	39	44
United States	25	37	38
European Union (28 countries)	31	42	55
OECD-Total	28	38	47
World	14	20	28

Source: OECD

Table 4: Results of steady state baseline simulation

d^*	Optimal debt to gdp ratio in steady state (no TFP)	40.16%
	Optimal debt ratio in steady state with 1% TFP	25.66%
X	Public private capital ratio	0.3023
γ	Economy *per capita* growth rate per period (no TFP)	0.88%
	Annual growth rate without any TFP	1.38%
γ^A	Annual growth rate with 1% TFP growth	2.39%
n	Number of children per person	1.25
	Age dependency ratio	28%

Table 5: Calibrated parameters: starting values (2013) and country specific values

	A_0 in 2013	*X=G/K* in 2013	Debt ratio in 2013(%GDP)	d^* in steady state, no TFP	d^* in steady state with TFP	d Adjustment to 2060 (%GDP)
USA	53.35	0.448	106.6	36.51	29.27	-77.23
UK	42.15	0.304	103.1	46.21	37.05	-66.05
France	42.94	0.370	116.1	45.36	36.38	-79.72
Germany	46.64	0.244	79.8	42.12	33.78	-46.02
Italy	32.96	0.336	141.4	59.09	47.39	-94.01
Belgium	29.74	0.194	106.7	65.49	52.52	-54.18
Netherlands	58.77	0.388	87.9	33.14	26.57	-61.13
Spain	43.55	0.356	111.5	44.72	36.24	-75.26
Japan	33.04	0.686	232.5	58.95	47.27	-185.23
Euro-zone	41.86	0.317	106.9	46.53	37.82	-69.58
OECD	48.50	0.417	111.2	40.16	32.20	-79.00

Source: Investment and Capital Stock Data Set 1960-2013, International Monetary Fund, October 2015; and National Accounts, OECD, Paris, May 2014

Table 6: Sensitivities to demographic parameters, OECD economies

n_λ	$\partial n/\partial \lambda$	0.2832
n_ε	$\partial n/\partial \varepsilon$	3.2809
n_ρ	$\partial n/\partial \rho$	-0.3098
d^*_λ	$\partial d^*/\partial \lambda$	0.0008
d^*_ε	$\partial d^*/\partial \varepsilon$	0.00932
d^*_ρ	$\partial d^*/\partial \rho$	-0.00088

Table 7: Subsidies as a policy tool

	Baseline	Decreased preference for children	Increased subsidy
Preference for children (ε)	0,30	0,23	0,23
Child-rearing subsidy (ρ_w)	0,15	0,15	0,32
Fertility rate	1,25	1,00	1,25
Total tax rate…	10,16%	9,38%	13,52%
…of which debt servicing	6,79%	6,83%	6,60%
…of which subsidy coverage	3,37%	2,55%	6,91%
Saving rate	7,93%	8,36%	8,36%
Interest rate (per period)	846,84%	846,23%	849,48%
Public-private capital ratio	30,23%	30,15%	30,54%
Government debt	35,69%	35,63%	35,95%
GDP growth (per period)	185,35%	184,70%	188,17%
GDP growth per capita (per period)	148,31%	184,70%	150,56%

Figure 1: The steady state growth-public debt possibility frontiers