Implications of Optimal Price Regulation in Sub-Prime Banking Markets

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Abstract

This paper investigates a model of endogenous product differentiation in the banking sector which incorporates both loan and depositor risks as well as increasing returns to scale (IRS), and Variable annual percentage rate (APR) lending behavior. The paper fills a gap in the literature which largely ignored IRS, Variable APRs and risks. Moreover, most of the generally empirical literature tends to assume product differentiation rather than obtain it as an endogenous choice. The main finding is that when an average cost pricing rule is imposed, banks will maximally differentiate their product. However, high quality-type banks benefit from increased market power, while low quality-type banks could benefit from either increased market share, or increased market power.

1 Introduction

A large body of literature has been devoted to product differentiation and competition in banking markets in recent years. See Barros (1997); Cohen and Mazzeo (2004); Degryse (1996); Kim, Kristiansen and Vale (2004); among others. However, the literature, especially that in the tradition of the New Empirical Industrial Organization (NEIO), has largely ignored two aspects of crucial importance in understanding banking markets. These aspects are risks and the widely posited increasing returns to scale (IRS) underlying banking activities.[1] Additionally, there is little attempt to address variable annual percentage rate (APR) lending behavior, which is very popular in mortgage, credit card and payday advances[2] lending behavior; see Flannery and Samolyk (2005), and the references therein.

Furthermore, there is a paucity of detailed studies addressing the effect of imposing Usury Laws, see Allen, Choi, and Fingerman (2004). Usury laws in this context simply refers to many states in the USA imposing maximum APRs that can be charged. For example, the maximum APR allowable in New York as of 2004 was 25 per cent. This paper seeks to fill the aforementioned gaps in the literature by developing a theory incorporating both risks and IRS in a model of endogenous product differentiation among banks engaging in variable APR lending behavior. The model can be interpreted as falling under the rubric of the *intermediation approach*.[3]

To be more specific the model is a two-stage game between duopolistic banks. Loan price competition, in the context of lines of credit, takes place in the second stage and a quality/type choice at the first stage. The model is motivated by the observation that banks operating under different charters face different levels of reserve requirements and, often times, on this basis, have distinct quality characteristics. For instance, the possible set of quality

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[1]In much of the earlier work, especially using 1980's data, only small banks (usually with less than $1 billion in deposits) were found to experience IRS; see Noulas, Ray and Miller (1990). However, according to more recent work, IRS is much more prevalent; see Bos and Kolari (2005) and the references therein.

[2]Of course, payday advances lending is done by alternative (non-bank) financial services entities.

[3]The other prevalent method in the banking literature is the *production approach*; see Noulas, Ray and Miller (1990).

characteristics could include: being foreign owned, being part of a larger holding company (suggesting 'deep pockets'), simply having more outlets across geographical space, offering a wider variety of services, considered too big to fail and/or being the leader in terms of the introduction of new services.

However, modeling multiple dimensions over which firms differ is a very complex issue in economics, and the standard approach is to create a single index which accounts for all these differences; in essence a hedonic interpretation. In our model, given the many dimensions over which banks could be dissimilar, rather than using quality we will refer to banks according to type. Type in this context is the single index capturing the multiplicity of differing characteristics. The type jargon is also appropriate, as in a more basic model, Thomas (2007), the single crossing property is a characteristic feature of the set-up to be presented. Here a bank type should be interpreted in a hedonic fashion in that the type represents a set of characteristics associated with such a bank. Therefore, for ease of exposition each quality/type choice will be identified on the basis of reserve requirement ratios.

As an example of identifying banks by reserve requirement ratios note that banks operating under different charters have historically distinct core businesses, leading to significantly different compositions of their overall portfolios[4]. But central banks levy reserve requirements on the basis of portfolio compositions. Consequently, banks operating under different charters face different levels of reserve requirements on average.[5]

Furthermore, to see how reserve requirements act as one of the lead indicators of bank type consider the following example: Prior to 1980 in the U.S., state sanctioned banks did not face a reserve requirement, while federally sanctioned banks did. Of course, it is usually the case that state sanctioned banks have a smaller capital base, and, obviously, do not have as wide a network across geographical space as federally sanctioned banks. Moreover, banks operating in a fractional reserve system will definitely have different

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[4]For example, thrift institutions and credit unions focus on savings deposits and mortgage lending; see Pulley, Berger and Humphrey (1993).

[5]In some countries, after 1980, banks have been levied with an explicitly different level of reserve requirements based on their charter. This was the case in Jamaica, for example, in the early 1990's where Commercial Banks, Merchant Banks and Building Societies faced different levels of reserve requirements.

risk structures based on their level of reserve requirements.

Addressing the issue of loan price competition raises the question of what are the inputs used to produce loans. Generally, there are three inputs: capital, labor and deposits. However, there is some controversy in the literature about whether deposits should be treated as inputs or outputs. In this paper deposits are treated as inputs and loans as outputs. This approach is consistent with Adams, Sickles and Roller (2000), (ASR), who integrated input and output markets, and treated deposits as inputs and loans as outputs.

ASR's method of bringing input and output markets together does not account for the relationship between deposits and loans against the background of reserve requirement ratios. Our particular specification integrates input (deposit) and output (loan) markets through reserve requirement ratios, an approach consistent with Sealey and Lindley (1977). This constitutes a more realistic framework within which the analysis can be performed.

The integration of input and output markets naturally brings to the fore the question of whether or not banks are price setters in both markets. The standard assumption in the literature is that banks act in a perfectly competitive deposit market, but that they can influence interest rates in the loan market (DeYoung and Hasan 1998, p. 575). This assumption can be justified by arguing that banks face competition in deposit markets from government and commercial paper, while they are the core source of loanable funds. An alternative to the perfectly competitive deposit market assumption is mark up pricing, which also makes loan interest rates a deterministic function of deposit interest rates.

The approach used in this paper is quite similar to mark-up pricing it does not take the standard form. Instead, *reserve requirement adjusted loan pricing* is used. Reserve requirement adjusted loan pricing posits that in the mark-up pricing equation the coefficient on $m.c.$ is $\frac{1}{(1-rr_t)}$, where rr_t is defined as the reserve requirement ratio of a bank of type t. The reserve requirement adjusted pricing rule is then a formalization of Usury Laws.

That is, the standard regulatory policy to protect consumers from unfair pricing, under IRS is to impose average cost pricing. As such, what we are positing is that the Usury Laws would adhere to this average cost pricing

regulation, hence generating the reserve requirement adjusted loan pricing rule adopted in the paper. In the discussion that follows reserve requirement loan pricing can also be considered consistent with government regulation of a natural monopoly, or firm characterized by IRS. That is, it amounts to average cost pricing. However, it does not imply zero economic profits because banks maximize expected profits as opposed to profits.

Furthermore, the literature on banking suggests that risks and increasing returns to scale (IRS) are important features of banking activities. In capturing risks and IRS in the model it should be noted that banks face risks both on the asset (loan) and liability (deposit) sides of their balance sheet (Carletti, Hartmann and Spagnolo, 2003; and De Bandt and Hartmann, 2000). We will model both types of risks and, through the inclusion of a risk premium, show IRS. Variable APRs are introduced through the risk premium, and are captured by banks offering low rates, referred to as *teaser rates*, premised on the assumption that risks are minimized.

Once borrowers make decisions on the basis of these teaser rates the banks then assess for risks and increase overall average rates to reflect those risks. One real world example of this is introductory APRs expiring with standard rates being assessed. Another example, is that standard and introductory APRs are replaced with default rates based on certain activities[6]. In both these examples late and other penalty fees serve to drive up effective APRs.

Our modeling framework is sufficiently compact such that modeling the risk premium also generates the IRS effects. The functional form that the risk premium takes is in the mold of Berger and Mester (1997) and Mester (1996). For further support of this formulation see Bos and Kolari (2005), p. 1575, indicating the finding of IRS regarding overall scale economies for a profit model. More compelling evidence of IRS is reported in Bos and Kolari (2005), p.1589. These authors, among many others, argue that IRS should not be represented independently of the risk coefficients in any model of banking. Now, IRS can be captured by explicitly specifying a convex production function or its properties can be mimicked by allowing input price to be a decreasing function of output. Our approach to capturing IRS takes the latter form, and will be explained in detail in later sections.

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[6]These activities include late payments, and "over the limit" adjustments among others.

Actually, a risk premium, $\theta(\ell_t)$, which is decreasing in output is attached to the deposit interest rate to generate the effect. In the intermediation approach, which is the method we employ, in addition to deposit interest rates, there are operating costs. Deposit interest rates are a second stage cost. Operating costs which consist of the costs of labor and capital are, however, a first stage cost in the context of our two-stage game. Operating costs are therefore considered fixed in the second stage. I will discuss the costs of labor and capital in detail when presenting aspects of the model where the first stage of the game becomes relevant.

Broadly speaking, the general conclusion of this paper is that banks will choose to maximally differentiate their products in order to soften price competition. However, different quality-types of banks may benefit from distinct sources of enhancing profits. On the one hand, the high quality bank benefits by strategically distinguishing its product from that of the low quality bank; thus gaining market power over its share of the market. To this end, the high quality bank also has an incentive to encourage the low quality bank to further distinguish itself from the high quality bank.

On the other hand, the low quality bank could benefit from either increased market power or from gaining additional market share. Benefiting from increased market share means that it would prefer to minimize the distance between itself and the high quality bank, provided it does not have to incur the additional costs of delivering a higher quality of service. However, in the presence of this cost maximal product differentiation prevails.

I further find that each type of bank advertises a teaser loan price which is an increasing function of the distance between types. The high quality-type charges a higher teaser loan interest rate than does the low quality type. The latter suggests that the difference in prices charged by the high quality bank and the low quality bank is an increasing function of the difference between bank types.

Interestingly, the solutions to the pricing equations have the same functional form as those in Kim, Kristiansen and Vale (2004) (KKV). But because of the inclusion of risks and IRS in this model, the actual solutions are quite different from those in KKV. The remainder of the paper is organized as follows: Section 2 outlines and develops the model. Section 3 provides a

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solution and discussion of the pricing equations—essentially the second stage solution of a two stage game. Comparative statics analysis is performed in section 4. Section 5 concludes, and Appendix A provides some examples illustrating the model results.

2 IRS, Risks and Product Differentiation

The model is premised on expected profit maximization by banks. I will adopt a consumer utility model that is widely used to generate demand for vertically differentiated products. In what follows we detail consumer behavior and then the second stage of our two-stage game of firm/bank behavior. That is, subsections 2.2 and 2.3 provide a discussion and formalize risks and IRS, before moving on to section 3 where each bank's objective function (expected profits) is outlined and the second stage of the game is solved.

2.1 Model Outline: Demand and the Game

Let n and N be defined as the total number of agents participating in the banking market, both loans and deposits; and the total number of agents participating in overall economic activity, respectively. Then, let $\nu \equiv N/n$; and let consumers be uniformly distributed on the closed interval $[0, n/N]$. The location $\hat{u}$ is interpreted as a preference parameter. More formally, $\hat{u} \sim \mathcal{U}ly[0, \frac{n}{N}]$. Therefore, the value of $\hat{u}$ is the fraction of consumers located to the left of $\hat{u}$ on the given interval. In essence, in larger banking markets the preference parameter is distributed over a wider space to reflect increased diversity in the market.

Also, a consumer either obtains a line of credit or not. In addition, assume that banks choose and announce an interest rate, r_t, which is the minimum value obtainable from their loan pricing rule. Borrowers then choose their creditor bank based on this minimum value of the interest rate. Some real world examples of this are credit card operations and adjustable rate mortgages. In the case of credit card companies they offer the lowest possible, standard (introductory), annual percentage rates (APRs); but, of course, they have default (standard) rates, which are usually higher than the announced rates. As for adjustable mortgage rates, they are usually adjusted upward. Finally, consumers are atomistic so that they can be treated as

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making their decisions given the types and output prices of both banks.

Let a bank type be denoted by t and suppose that in a banking duopoly, the first stage choices are such that $t\epsilon\{L, H\}$, that is, one bank chooses H and the other bank chooses L, where both L and H are numbers on the interval $[0, \frac{n}{N}]$. Without loss of generality, $L \leq H$ so that H is the "high type" and L is the "low type". Then a consumer of type $\hat{u}$ has the following payoff function:

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\begin{cases}
\hat{u}\nu H - r_H & \text{when buying from type } H; \\
\hat{u}\nu L - r_L & \text{when buying from type } L; \\
0 & \text{when not buying}
\end{cases}
$$

Where, again, r_t is a bank of type t's scheduled minimum loan interest rate, which will also be referred to as the "teaser" rate. Here, because of the risk return trade-off, ν increases the payoff to consumers who take a loan. That is, in economies where the loan market is not very well developed, ν is large, consumers who acquire loans to invest take on more risk than consumers in well established loan markets, where ν is relatively small.[7] Now, note formally that in order to induce new participants in the loan market as the economy grows, then, the payoff $\hat{u}\nu t - r_t$ must be increased as output increases. It should be clear that for given r_t, $\hat{u}$ has to be increased to generate this effect since ν, after the fact, will be reduced.

To see that the macro economy and banking markets are connected in the model assume that N is fixed, and recall that $\hat{u}$ simply represents the fraction of consumers to the left of $\hat{u}$ on the line segment. Now, those consumers who did not participate in the loan market must have perceived the return from banking activities to be negative. But as the macro-economy improves the utility from loans to some of these said consumers becomes positive, because $\hat{u}$ is increased, and they participate in the loan market. Simply put, n is increased, or equivalently, ν is reduced. Effectively, what happens in the model is that n is a proxy for overall output in the economy. Thus, in the

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[7]Another way to view this is as a kind of convergence similar to that discussed in macro-economic growth theory. Relatively, smaller loan markets yielding higher returns should increase agent participation, hence, *ceteris paribus*, all markets should eventually have similar ν.

formal model changes in macroeconomic output are reflected through changes in n.

Here, we can treat N as fixed, or in such a way that all new participants in economic activity, especially investors from abroad, participate in the banking market. As such, during economic booms when investment projects are more profitable agents generate higher payoff, $\hat{u}$, from borrowing. Consequently, n grows because more locals and foreign interests participate in banking activities, whereas N grows only due to foreign interests.

Overall, then, n, grows faster than N, thus having the same qualitative implication of n growing and N being fixed. The decrease in ν referred to here will come about because as discussed in section 2.2 investment projects are more profitable during booms. This amounts to saying that the consumer benefit from taking a loan, $\hat{u}$, is increased, and is what drives down ν. Of course, this is a counter balancing force where the growth in $\hat{u}$ is counteracted by the reduction in ν to ensure that at some point it would not be beneficial for consumers to seek more loans.

Now, solving the consumer's problem by finding the consumer u indifferent between buying from the high type or low type yields[8]:

$$\hat{u}\nu H - r_H = \hat{u}\nu L - r_L \iff u = \frac{(r_H - r_L)}{\nu(H-L)} \geq 0. \tag{1}$$

Now, let $U \equiv \nu u \, \epsilon [0,1]$. Therefore we can write each firm's demand, denoted by $\ell_t \equiv D_t(r_H, r_L)$. as follows:

$$\ell_H \equiv D_H(r_H, r_L) = \begin{cases} 0 & \text{if } U \geq 1 \\ \frac{n}{N}(1-U) & \text{if } 0 \leq U \leq 1 \\ \frac{n}{N} & \text{if } U \leq 0 \end{cases}$$

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[8]In solving the model only cases where both firms have positive market share are considered. Note that if any firm has zero market share then only one firm exists and we can no longer discuss product differentiation. Furthermore, we set the utility of consuming the outside good to zero.

$$\ell_L \equiv D_L(r_H, r_L) = \begin{cases} \frac{n}{N} & \text{if } U \geq 1 \\ \frac{n}{N}U & \text{if } 0 \leq U \leq 1 \\ 0 & \text{if } U \leq 0 \end{cases}$$

In the model of firm behavior firms choose their type in the first stage and compete in prices in the second stage. Clearly then, as will be discussed in later sections, in stage 2 types are exogenous variables which will affect prices as in standard models. Now, to formalize the complete model consider the following duopoly game:

Stage 1: Each bank chooses its quality-type, $t \in [\underline{t}, \bar{t}] \subset \Re_+$.

Stage 2: Each bank chooses its price, $r_t \in \Re_+$, given its own type and that of the other firm.

A bank of type t then maximizes expected profits, $\Pi(\alpha, \gamma, \theta, r_t, r_{-t}) \equiv$ expected revenue - expected costs, in each stage, given the other bank's choices. Here α and γ are risk coefficients, θ is the risk premium, and r_{-t} is the loan interest rate chosen by the other bank. That is, the equilibrium concept being applied is that of a (subgame perfect) Nash Equilibrium. Deposit interest rates are not explicitly a part of the expected profit function due to the reserve requirement adjusted loan pricing assumption.

The idea of reserve requirement adjusted loan pricing is formally captured as follows: Let $W_t \equiv \theta(\ell)\varpi_t$ be defined as the risk adjusted deposit interest rate for a bank of type t, where $\theta(\ell)$ is the risk premium and ϖ_t is the deposit interest rate when risks are at their lowest. Let $R_t \equiv \theta(\ell)r_t$ be the risk adjusted loan interest rate[9] for a bank of type t, and rr_t be defined as the reserve requirement ratio of a bank of type t. Since, as we show later, $\theta(\ell)$ takes a minimum value of one, then when $\theta(\ell)$ is at its minimum, $R_t = r_t$. Now, reserve requirement adjusted loan pricing simply posits the loan pricing rule $R_t = \frac{W_t}{1-rr_t}$, or equivalently, $r_t = \frac{\varpi_t}{1-rr_t}$; hence the given name.

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[9]Both the risk adjusted deposit and loan rate should be viewed as the average risk adjusted price per unit, since not all individuals will default. However, banks can treat these prices as they appear in section 3.

Here one should take a keen interest in noting that each bank's choice variable in the second stage is r_t. ϖ_t could be viewed as the choice variable, but the solution is invariant to the choice of r_t or ϖ_t. This is so because in the second stage, according to the reserve requirement adjusted loan pricing rule, r_t is equal to ϖ_t times the constant $\frac{1}{1-rr_t}$. Reserve requirement is constant because it does not vary with price, but only with type, thus, the invariance of the solution to these two choice variables. Also, once r_t is chosen, the demand functions outlined above will give equilibrium quantity and automatically R_t will be determined since it is only a function of the variables r_t and $\theta(\ell)$, which itself is clearly dependent on loan quantity. Note also that in the optimization problem banks cannot choose R_t because that requires setting both price and quantity; a situation that no market structure affords any firm.

Before proceeding with solving the game some structure is added to derive the firms' payoff functions explicitly[10]. Arranging the banks, according to types, in ascending order, posit $t_1 = H > t_2 = L$. It will be shown that high type banks charge a higher price than low type banks. That is, $r_1 = r_H > r_2 = r_L$ in equilibrium, a reflection of the high type bank's superior product. Note that any other equilibrium outcome is not reasonable, because it would mean that acquiring the service from the high type bank would be a strictly dominant strategy for consumers/borrowers. Thus, firms have stage 2 expected profits which can be described in more detail after dealing with some additional issues, namely risk and IRS.

2.2   Discussion of Risks and Increasing Returns to Scale

An assumption of our model is that the risk premium is decreasing in loan quantity. This seems to suggest that the marginal consumer is the best risk, however, this is not the case. As proof the change in risks are derived indirectly from the change in r_t in this case. Consequently, the relevant quantity to be examined is $\partial\theta/\partial\ell_t$ times $\partial\ell_t/\partial r_t$. With the risk premium decreasing in loan quantity, and a downward sloping demand function, $\partial\ell_t/\partial r_t < 0$, the overall effect of a movement along the demand curve is an inverse relationship. That is a reduction in r_t leads to an increase in the risk premium.

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[10]So far the discussion of the model has followed closely that outlined in KKV.

Obviously, the marginal consumer is not the best in terms of risk.

The remainder of this section is now dedicated to discussing and formalizing the risk premium, θ, which is in accordance with arguments presented by Berger and Mester (1997). Another characteristic is that the premium will also have a minimum value of one, and since it is decreasing in loan output, this would occur at the point where this output is at its maximum. One should also remain keen about the fact that the risk premium, based on this framework, is decreasing in a bank's own loan rate and decreasing in output.

In this subsection one of the ideas we want to establish is that both a bank's costs and revenues are stochastic. To develop this idea we stress that there are two distinct types of risks faced by a banking firm. We will refer to the risks on the liability side as *depositor risks*; and that on the asset side as *loan risks*. Depositor risks are made up of two distinct components, systemic and systematic risks; and liquidity risks[11]. However, the term depositor risks is sufficient because the germane idea we want to drive home is that a bank's costs are stochastic. On the revenue/asset side, risks are constituted by borrowers defaulting on loans—thus the term loan risks.

Now, it is reasonable to expect that n increases during booms, and, assuming that N is fixed would immediately result in pro-cyclical loan behavior. In the banking literature there is evidence that banks tend to fail (because of systemic, systematic and/or liquidity risks) during periods of economic downturn (see De Bandt and Hartmann, 2000). Investment projects, in general, also tend to fail during periods of economic downturn, hence triggering higher default rates among borrowers. As a result we posit that both perceived depositor and perceived loan risks are inversely related to total output in the economy[12].

But both deposits and loans are proportional to output, and, thus, will also be inversely related to their associated risks. Output and deposits are proportional because, assuming a constant marginal propensity to save and that agents in the economy do not shift away from holding deposits at banks dur-

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[11]See Carletti, Hartmann and Spagnolo, 2003; and De Bandt and Hartmann, 2000).

[12]Of course, beyond the full employment level of output this relationship could be reversed as the economy overheats. Since agents can predict this we assume that they are fully protected against it and rule it out of the analysis.

ing economic booms and vice versa, deposits at banks will increase as overall output in the economy increases. Ceteris paribus, as overall deposits increase each bank's deposits will also increase; thus, we can claim that as individual deposits increase each bank's depositor risk falls. In effect, what occurs is that the probability of a bad event/depositor risks becomes endogenously determined.

Similarly, since investments tend to yield higher rates of return during economic booms there will be more overall loans during booms. Thus, output is also proportional to total loans in the economy. But if total loans are increasing we should reasonably expect that each bank's loans are also increasing. Accordingly, we can argue that as a bank's own loans increase its loan risk will fall. Here we highlight the effect of fewer new lines of credit outweighing the effect of higher rates of utilization of existing lines of credit during recessionary periods; and vice versa.

Despite widespread discussion of increasing returns to scale (IRS) in the banking literature (see Bos and Kolari (2005)), especially with regard to efficiency, little attempt has been made to explicitly model this phenomenon in the context of banking product differentiation. Our way of capturing IRS works through two channels, each causing an outward shift in the supply of deposits curve, thus resulting in a reduction of deposit rates[13].

Firstly, an increase in an economy's output directly results in more savings, and, therefore, more deposits. Secondly, this same increase in output, through a reduction in depositor risks, indirectly causes an increase in the supply of deposits. To formalize these arguments, the idea that depositor risks are inversely related to overall deposits of an individual bank is used to introduce a risk premium on deposits which is decreasing in a bank's own deposits. Since deposits and loans are positively related, as will be seen from the intermediation function introduced later, we generate increasing returns to scale (IRS) through decreasing input price as output (loans) increases[14].

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[13]For an example of a quality and risk adjusted cost function in the banking literature see Hughes and Mester (1993).

[14]Actually IRS properties are mimicked by having declining input costs and a linear production technology, rather than the usual convex production technology that would be expected to reflect IRS.

2.3 Formally Modeling Risks and IRS

Recall and consider the following definitions in the second stage of the game: Denote profits of each bank/firm by Π_t. Let the respective risk adjusted and unadjusted average cost be $W_t \equiv \theta \varpi_t$ and $w_t \equiv \frac{W_t}{1-rr_t}$. Furthermore, ℓ_t and d_t denote total loans and total deposits of bank t, respectively. The banks' intermediation/production function simply says that banks can only make loans with the fraction of deposits that are not held as required reserves: $\ell_t = (1 - rr_t)d_t$. Recall that rr_t is defined as bank t's reserve requirement ratio. Moreover, let α_t be the fraction of customers/borrowers that a bank of type t perceives will not default on loans. Further, assume that banks are risk neutral so that the distribution of defaults is immaterial. Let γ_t be defined as the fraction of banks of type t, that depositors perceive will not default.

Now, formalizing the discussion of risks let $e^{-\delta d_t}$ and $e^{-\sigma \ell_t}$ be defined as the perceived depositor risk and perceived loan risk, respectively. Here $\delta \epsilon (0, 1)$ and $\sigma \epsilon (0, 1)$ are defined as the respective quantity weighted (divided by quantity) absolute values of the elasticities of depositor risks with respect to deposits, and loan risks with respect to loans[15]. Also, let depositor risks be greater than loan risks in actual magnitude, such that $\sigma > \frac{\delta e}{(1-rr_t)}$. This addresses the issue of portfolio alignment and simply says that a bank will not allow its risks exposures due to loans to be greater than that which its depositors face. Now, define the risk coefficients such that $\gamma_t = 1 - e^{-\delta d_t}$ and substituting from the intermediation function yields $\gamma_t = 1 - e^{\frac{-\delta \ell_t}{(1-rr_t)}}$. And, finally, $\alpha_t = 1 - e^{-\sigma \ell_t}$. Now formally specify the risk premium θ, such that

$$\theta \equiv \left[\frac{e^{-\frac{\delta}{(1-rr_t)}} - e^{-\sigma}}{e^{-\frac{\delta \ell_t}{(1-rr_t)}} - e^{-\sigma \ell_t}} \right]$$

This specification of the risk premium is the ratio of the difference in the overall economy's depositor and loan risks (the numerator) to the difference in an individual bank's depositor and loan risks (the denominator). Note that it is a function of the risk coefficients as per the suggestions of Berger and

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[15]It is a very simple exercise to check these definitions. Simply calculate these elasticities to verify the statements.

Mester. Also, generalize to make $\varpi_t(\ell_t)$ a decreasing function of ℓ_t, hence $\varpi_t{\prime}(\ell_t) < 0$ indicating that deposit interest rates and loans are inversely related. Even with this generalization under the assumption $\sigma > \frac{\delta e}{(1-rr_t)}$ derivatives show that θ is decreasing in ℓ_t and in fact has minimum value one when ℓ_t, a fractional quantity, equals one[16]. Thus the risk adjusted rate and the unadjusted rate would be equal at this point. Hence, the price that borrowers choose is consistent with the banks' loan pricing rule.

Earlier we alluded to the marginal consumer not being the best in terms of risk for clarity we show this here. Recall ℓ_t, and for emphasis indicate $\ell_t(r_t)$, where $\ell_t{\prime}(r_t) < 0$. Of course, in this situation we are dealing with $\theta(\ell_t)$, and, consequently, $w_t(\ell_t)$. Now, since we are positing that ℓ_t changes only with r_t to perform a similar calculation to that immediately above we differentiate w_t with respect to r_t implying $\frac{\partial w_t}{\partial r_t}$ is:

$$\frac{\partial \theta}{\partial \ell_t}\frac{\partial \ell_t}{\partial r_t}\frac{\varpi_t}{1 - rr_t} + \theta\frac{\varpi_t{\prime}(l_t)}{1 - rr_t}\frac{\partial \ell_t}{\partial r_t} > 0$$

The inequality constraint is true because $\frac{\partial \theta}{\partial \ell_t}$, $\frac{\partial \ell_t}{\partial r_t}$, $\varpi_t{\prime}(l_t) < 0$, with all other quantities above positive. This shows that the marginal consumer is not the best in terms of risk. The IRS result can now be shown. Recall, $w_t \equiv \theta\frac{\varpi_t}{1-rr_t}$, hence each bank's cost function is: $C_t(\ell_t) = w_t\ell_t$

Recall that IRS implies that marginal costs, (mc), are less than average costs (ac). Consequently, the ratio mc/ac should be less than one for IRS to exists. This can now be shown from the cost function. Substituting w_t in the cost function yields :

$$C_t(\ell_t) = \left[\frac{e^{-\frac{\delta}{(1-rr_t)}} - e^{-\sigma}}{e^{-\frac{\delta \ell_t}{(1-rr_t)}} - e^{-\sigma \ell_t}} \right] \frac{\varpi_t}{1 - rr_t}\ell_t$$

from which ac is:

$$\left[\frac{e^{-\frac{\delta}{(1-rr_t)}} - e^{-\sigma}}{e^{-\frac{\delta \ell_t}{(1-rr_t)}} - e^{-\sigma \ell_t}} \right] \frac{\varpi_t}{1 - rr_t}$$

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[16]Acknowledging that n proxies overall output in the economy, we could also show this by using the chain rule to find partial derivative of θ with respect to n.

and mc is:

$$\frac{\theta \varpi_t}{1-rr_t} - \left[\frac{\left(e^{-\frac{\delta}{(1-rr_t)}} - e^{-\sigma}\right)\left(\sigma e^{-\sigma \ell_t} - \frac{\delta}{(1-rr_t)}e^{-\frac{\delta \ell_t}{(1-rr_t)}}\right)}{\left(e^{-\frac{\delta \ell_t}{(1-rr_t)}} - e^{-\sigma \ell_t}\right)^2}\right]\frac{\varpi_t}{1-rr_t}\ell_t + \frac{\theta \ell_t \varpi_t\prime(\ell_t)}{1-rr_t}$$

Hence one can deduce that

$$\frac{mc}{ac} = 1 - \left[\frac{\left(e^{-\frac{\delta}{(1-rr_t)}} - e^{-\sigma}\right)\left(\sigma e^{-\sigma \ell_t} - \frac{\delta}{(1-rr_t)}e^{-\frac{\delta \ell_t}{(1-rr_t)}}\right)}{\left(e^{-\frac{\delta \ell_t}{(1-rr_t)}} - e^{-\sigma \ell_t}\right)^2}\right]\frac{\ell_t}{\theta} + \frac{\theta \ell_t \varpi_t\prime(\ell_t)}{1-rr_t}$$

But the term

$$\left[\frac{\left(e^{-\frac{\delta}{(1-rr_t)}} - e^{-\sigma}\right)\left(\sigma e^{-\sigma \ell_t} - \frac{\delta}{(1-rr_t)}e^{-\frac{\delta \ell_t}{(1-rr_t)}}\right)}{\left(e^{-\frac{\delta \ell_t}{(1-rr_t)}} - e^{-\sigma \ell_t}\right)^2}\right]\frac{\ell_t}{\theta} > 0$$

This is true because $\sigma > \frac{\delta e}{(1-rr_t)}$ implies that all the terms contained therein are positive. And to reiterate $\varpi_t\prime(\ell_t) < 0$, hence $\frac{mc}{ac} < 1$ and therefore it is shown that this structure exhibits IRS.

3   Second Stage Solution: Interest Rates

Upon reverting to the bank's optimization problem, the following is a useful note. It should be obvious that the only rational loan pricing rules are ones such that $R_t \geq \frac{W_t}{(1-rr_t)}$. Now at the minimum, that is at $R_t = \frac{W_t}{(1-rr_t)}$ one has a special case which is what we termed *reserve requirement adjusted loan pricing*. Reiterating that banks act prudently, and incorporate the risk adjusted rate in representing profits we can specify the banks' objective functions. Furthermore, banks must incorporate all risks, including depositor risks in calculating expected profits[17].

A full specification of expected profits is: $\Pi_t = \alpha_t R_t \ell_t - \gamma_t W_t d_t$ which the bank will maximize with respect to the unadjusted rate of interest, r_t, subject to:

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[17]See Carletti, Hartmann and Spagnolo (2003); and De Bandt and Hartmann (2000).

$$(i)\ell_t = (1-rr_t)d_t; \ (ii)\alpha_t = (1-e^{-\sigma\ell_t}); \ (iii)\gamma_t = (1-e^{-\delta d_t}); \ (iv)R_t = \frac{W_t}{(1-rr_t)}$$

As was previously discussed, maximization is with respect to the unadjusted rate of interest, r_t. Again, note that R_t is untenable as the choice variable because it implies that the bank can choose both quantity and price. Actually, a determination of r_t is sufficient to determine quantity from the demand functions, and, thus, R_t will follow naturally.

The constraint (i) is simply the banks intermediation function. This is so because barring other sources of funding loans, and/or assuming that a bank's main role is intermediation, that is, taking deposits to finance loans, then total loans must be less than or equal to $(1-rr_t)$ times total deposits. Of course, since we are assuming that the bank's objective is to maximize profits we assume that we are always on the boundary of the inequality constraint, hence the strict equality. Then, substituting the constraints and definitions into the profit function gives:

$$\Pi_t = \left[(1-e^{-\sigma\ell_t})\theta r_t - (1-e^{-\frac{\delta\ell_t}{(1-rr_t)}})\frac{\theta\varpi_t}{(1-rr_t)}\right]\ell_t$$

Recalling $r_t = \frac{\varpi_t}{(1-rr_t)}$ yields: $\Pi_t = \theta\left[(1-e^{-\sigma\ell_t})r_t - (1-e^{-\frac{\delta\ell_t}{(1-rr_t)}})r_t\right]\ell_t$

Simplifying and substituting for θ gives:

$$\Pi_t = \left[\frac{e^{-\frac{\delta}{(1-rr_t)}} - e^{-\sigma}}{e^{-\frac{\delta\ell_t}{(1-rr_t)}} - e^{-\sigma\ell_t}}\right]\left[e^{-\frac{\delta\ell_t}{(1-rr_t)}} - e^{-\sigma\ell_t}\right]r_t\ell_t$$

Which we further simplify to get:

$$\Pi_t = [e^{-\frac{\delta}{(1-rr_t)}} - e^{-\sigma}]r_t\ell_t \tag{2}$$

Now to solve the problem of each of the types recall their specific demand functions and substitute into the profit function. Therefore for the high type

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firm the profit function is[18] :

$$\max_{r_H} \Pi_H \ = [e^{-\frac{\delta}{(1-rr_H)}} - e^{-\sigma}]r_H l_H \tag{3}$$

Similarly, the low type firm's profit function is:

$$\max_{r_L} \Pi_L \ = [e^{-\frac{\delta}{(1-rr_L)}} - e^{-\sigma}]r_L l_L \tag{4}$$

Then to solve for price, note that the corner solutions imply zero profit but that interior solutions will generate positive profit, therefore the first order conditions (FOCs) give the solution and imply:

$$r_H \ = \frac{2}{3}(H - L) \tag{5}$$

Hence:

$$r_L \ = \frac{1}{3}(H - L) \tag{6}$$

From this set up the following propositions hold:

Proposition 1: *The high quality-type bank will necessarily advertise a higher teaser rate—minimum value for loan interest rates— than will the low quality-type bank.*

Proposition 2: *As bank types become more distinct banks of each type will be able to advertise higher prices. That is, the more distinct banks are in terms of type, for example, high type vs. low type banks, then competition will be less intense.*

Proposition 3: *The wedge between the teaser rate/interest rate advertised by high quality-types and low-quality types is an increasing function of the distance between the different bank types.*

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[18]Here and in the remainder of the paper we use U, since the inclusion of ν or its inverse simply carries along a constant which is immaterial to the solution of the optimization problem.

Proof of Propositions :

(1) The proof of proposition 1 is straight forward from the following equation derived from equations (5) and (6).

$$r_H - r_L = \frac{(H-L)}{3} > 0 \tag{7}$$

(2) Proposition 2's proof is: $\frac{\partial r_H}{\partial (H-L)}, \frac{\partial r_L}{\partial (H-L)} \geq 0$; *or,* $\frac{\partial r_t}{\partial (H-L)} \geq 0, t \in \{H, L\};$. These results follow directly from equations (5) and (6).

(3) Proposition 3 follows from the fact that: $\frac{\partial (r_H - r_L)}{\partial (H-L)} \geq 0, t \in \{H, L\}$. Again, the results are derived from equations (5) and (6).

4   Product Differentiation:

In this section it becomes necessary to say explicitly how the reserve requirement ratio of each type of bank, rr_t, is related to that type. To this end, and noting that rr_t is a fraction, recall $t\epsilon[\underline{t}, \bar{t}]$ and let $rr_t = \frac{t-\underline{t}}{t}$.

Also, to avoid the pathological circumstance that banks keep all deposits with the central bank as reserves, posit that $\underline{t}$ is bounded away from zero. Note that rr_t is an increasing function in t, and since reserve requirements is a tax on banks then it implies that to be a higher quality bank one must incur a higher cost in the form of this tax. Furthermore, let operating costs be denoted by $F(t) \equiv f(k(t) + s(t))$. Here $k(t)$ and $s(t)$ represent, respectively, the cost of capital and the cost of the labor skills[19] of employees as a function of type. Also, let $f'(\cdot) > 0$. Of course, by the Inverse Function Theorem (IFT) $F^{-1}(\cdot)$ exists and the individual functions $k(t)$ and $s(t)$ can be recovered.

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[19]More highly skilled managers, loan officers, tellers and service representatives among other employees will ultimately contribute to the type of institution that a bank performs as.

Additionally, defining weights on the degree of product differentiation and on the difference in the probability that the banks default and that its borrowers default as $2\delta e^{-\frac{\delta t}{t}}$ and $\underline{t}$ respectively, let $A_t \equiv e^{-\frac{\delta}{(1-rrt)}} - e^{-\sigma}$, such that $A_H > 3(H-L)\nu F'(H)$. Finally, let $Z_t \equiv (H-L)\delta e^{-\frac{\delta t}{t}}$. With this established one can proceed to analyze the model. The analysis is as follows: Note from the pricing solutions above that $U = \frac{1}{3}$ and that H and L are exogenous in the second stage. Accordingly, the following assumptions are entirely in terms of exogenous parameters in the second stage:

$$(A1) \ \ \frac{1}{1+\frac{\underline{t}A_t}{Z_t}} < U < \frac{2}{3}; \ \ (A2) \ \ \varpi_t(t) < \underline{t}U$$

All $(A1)$ says is that both firms will have positive market share since both A_t and Z_t are positive. Further, $(A1)$ implies that:

$$\frac{1}{1+\frac{\underline{t}A_t}{Z_t}} < \frac{1}{3} \Leftrightarrow 3 < 1 + \frac{\underline{t}A_t}{Z_t} \Leftrightarrow 2 < \frac{\underline{t}A_t}{Z_t} \Leftrightarrow 2Z_t < \underline{t}A_t$$

Then by the definition of Z_t and A_t assumption $(A1)$ intuitively says that the weighted degree of product differentiation must be small (not too large) relative to the weighted difference in the probability that the banks default and that their borrowers default. Under the assumptions, $(A1)$ and $(A2)$, the following theorem and set of propositions which support it are true:

Theorem 1: *Under assumptions $(A1)$ and $(A2)$, the principle of maximal product differentiation is sustained.*

The proof of theorem 1 will be broken down into a statement of four propositions, being propositions (4), (5), (6) and (7). The proof of proposition (5) is found in equations (8) and (9), which shows that under assumption $(A1)$, there is an incentive for a bank of a particular type to distinguish itself from the other type. As a matter of fact, the optimal result is for there to be maximal product differentiation. Correspondingly, the proof of proposition (6) is found in equation (10), where under $(A2)$ it is shown that high type banks benefit from the low type trying to further distinguish itself from the high type. This result is consistent with a benefit accruing to the high type as the low type bank differentiates its product; a circumstance which reinforces

the maximal product differentiation result. In contrast, under $(A2)$ the low quality-type bank may suffer a loss as the high quality-type bank further distinguishes itself from the low type. This result appears in equation (11).

Theorem 1: *Under assumptions $(A1)$ and $(A2)$, the principle of maximal product differentiation is sustained.*

Proposition 4: *Under $(A1)$ there is a unique subgame perfect Nash equilibrium. In this equilibrium the two banks choose different types, $t = \{L, H\}$.*

Proposition 5: *Under assumption $(A1)$, bank profitability is increasing in the distance between types.*

Proposition 6: *Under assumption $(A1)$, high type profitability is increasing as L falls. .*

Proposition 7: *Under assumptions $(A1)$ and $(A2)$, low type profitability is inversely related to H.*

Of course, there seems to be an apparent contradiction between propositions 5,6 and 7. This apparent contradiction between propositions 5, 6 and 7 is rather insightful about which of two countervailing forces is preferred by either type. These two forces are highlighted in Tirole (1988), who points out that in these models there are two off-setting forces that firms weigh against each other in making their optimal decision. These two forces are: the strategic market power gained by distinguishing one's product from that of a competitor; and, the market share gained by coming closer to a rival. An argument to demonstrate why the apparent contradiction is less real than apparent follows. Also, this argument shows that rather than confounding, the seemingly contradictory result clarifies the workings of the model.

As already stated propositions 5 and 6 reinforce each other. However, proposition 7 at a cursory look seems to contradict both. There is actually no contradiction because the conclusion of proposition 5 holds and the high type bank will continue to benefit from increasing its own quality, and, therefore, maximal product differentiation is upheld. The idea is that the low quality type, even though it would prefer to minimize the distance between itself and the high quality type, as per proposition 7, does not find it optimal to do so

if this means that the low type has to incur the cost of providing a higher quality product.

Therefore, provided that it does not have to incur the cost of higher quality, the low quality-type bank benefits more from increased market share. Conversely, the high quality type bank benefits more by strategically distinguishing itself from its rival. The formal arguments supporting this claim and the proof of propositions 4, 5, 6 and 7 are now outlined. From standard comparative statics results the effects on profits of exogenous changes in types are as follows:

Proofs of Proposition 4 and 5:

High Type Banks:

Now Recall:

$$\Pi_H \qquad = [e^{-\frac{\delta}{(1-rr_H)}} - e^{-\sigma}]r_H l_H - F(H)$$

and that[20]:

$$rr_t = \frac{t - \underline{t}}{t}$$

Using the definition of A_t and comparative statics[21] this implies:

$$\Pi_H \qquad = [e^{-\frac{\delta H}{\underline{t}}} - e^{-\sigma}]r_H l_H - F(H)$$

$$\frac{\partial \Pi_H}{\partial H} : \quad r_H \left\{ \frac{\partial A_H}{\partial H} l_H + A_H \left[\frac{\partial l_H}{\partial H} + \frac{\partial l_H}{\partial r_L} \frac{\partial r_L}{\partial H} \right] \right\} - F'(H)$$

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[20] Note that the specification of rr_t requires that $\underline{t}$ be bounded away from zero, because otherwise one may be left with the pathological circumstance that $rr_t = 1$ always. In which case, the only business banks undertake is to keep all their deposits at the central bank as reserves. This is rather unrealistic and so is ruled out.

[21] Recall that with comparative statics derivatives need not be taken with respect to the choice variable.

But

$$\frac{\partial A_H}{\partial H} = -\frac{\delta e^{-\frac{\delta H}{t}}}{\underline{t}}; \frac{\partial l_H}{\partial H} = \frac{r_H - r_L}{\nu(H-L)^2}; \frac{\partial l_H}{\partial r_L} = \frac{1}{\nu(H-L)}; \frac{\partial r_L}{\partial H} = \frac{1}{3}$$

And recall:

$$U = \frac{r_H - r_L}{(H-L)} \Rightarrow l_H = \frac{(1-U)}{\nu}$$

Therefore:

$$\frac{\partial \Pi_H}{\partial H} : \quad r_H \left\{ -\frac{\delta e^{-\frac{\delta H}{t}}}{\underline{t}} \frac{(1-U)}{\nu} + A_H \left[\frac{U}{\nu(H-L)} + \frac{1}{3\nu(H-L)} \right] \right\} - F'(H)$$

Since $A_H, U \, and \, (H-L)$ are all positive then maximal product differentiation, which requires that $\frac{\partial \Pi_H}{\partial H} > 0$, implies:

$$A_H \left[\frac{U}{(H-L)} + \frac{1}{3(H-L)} \right] > (1-U)\frac{\delta e^{-\frac{\delta H}{t}}}{\underline{t}} + \nu F'(H)$$

$\Leftrightarrow$

$$3U A_H \underline{t} + \underline{t} A_H > 3(H-L)(1-U)\delta e^{-\frac{\delta H}{t}} + 3(H-L)\underline{t}\nu F'(H)$$

$\Leftrightarrow$

$$3U A_H \underline{t} + \underline{t}[A_H - 3(H-L)\nu F'(H)] > 3(H-L)(1-U)\delta e^{-\frac{\delta H}{t}}$$

Therefore under the assumption that $A_H > 3(H-L)\nu F'(H)$ the following, after substituting for A_H, is a sufficient condition to generate the maximal product differentiation result:

$$3U\underline{t}[e^{-\frac{\delta H}{t}} - e^{-\sigma}] > 3(H-L)(1-U)\delta e^{-\frac{\delta H}{t}}$$

$\Leftrightarrow$

$$3\underline{t} \left\{ U \left[e^{-\frac{\delta H}{t}} - e^{-\sigma} + \frac{(H-L)\delta e^{-\frac{\delta H}{t}}}{\underline{t}} \right] - \frac{(H-L)\delta e^{-\frac{\delta H}{t}}}{\underline{t}} \right\} > 0$$

In which case it is only necessary to have the term in curly brackets greater than zero. This implies (recall the definitions of A_t and Z_t):

$$U > \frac{(H-L)\delta e^{-\frac{\delta H}{\underline{t}}}}{\underline{t}A_H + (H-L)\delta e^{-\frac{\delta H}{\underline{t}}}} = \frac{1}{1 + \frac{\underline{t}A_H}{Z_H}} \qquad (8)$$

Low Type Banks:

$$\Pi_L \qquad = [e^{-\frac{\delta}{(1-rr_L)}} - e^{-\sigma}]r_L l_L - F(L)$$

Using comparative statics combined with the definitions of A_t and rr_t, this implies:

$$\Pi_L \qquad = [e^{-\frac{\delta_L}{\underline{t}}} - e^{-\sigma}]r_L l_L - F(L)$$

$$\frac{\partial \Pi_L}{\partial L} : \quad r_L \left\{ \frac{\partial A_L}{\partial L} l_L + A_L \left[\frac{\partial l_L}{\partial L} + \frac{\partial l_L}{\partial r_L} \frac{\partial r_L}{\partial L} \right] \right\} - F'(L)$$

But

$$\frac{\partial A_L}{\partial L} = -\frac{\delta e^{-\frac{\delta_L}{\underline{t}}}}{\underline{t}} ; \frac{\partial l_L}{\partial L} = \frac{r_H - r_L}{\nu(H-L)^2} ; \frac{\partial l_L}{\partial r_L} = \frac{1}{\nu(H-L)} ; \frac{\partial r_L}{\partial L} = -\frac{2}{3}$$

substituting U appropriately gives:

$$\frac{\partial \Pi_L}{\partial L} : \quad r_L \left\{ -\frac{\delta U e^{-\frac{\delta_L}{\underline{t}}}}{\nu \underline{t}} + A_L \left[\frac{U}{\nu(H-L)} - \frac{2}{3\nu(H-L)} \right] \right\} - F'(L)$$

Since A_L, U and $(H - L)$ are all positive then a sufficient condition for maximal product differentiation, which requires that $\frac{\partial \Pi_L}{\partial L} < 0$, is:

$$A_L \left[\frac{U}{\nu(H-L)} - \frac{2}{3\nu(H-L)} \right] < 0$$

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$$\Leftrightarrow$$

$$3U - 2 < 0 \Leftrightarrow U < \frac{2}{3} \tag{9}$$

Proof of Proposition 6:

$$\Pi_H = [e^{-\frac{\delta H}{t}} - e^{-\sigma}]r_H l_H - F(H)$$

Recall A_t and performing comparative statics, to get:

$$\frac{\partial \Pi_H}{\partial L} = r_H A_H \frac{\partial l_H}{\partial L}$$

Recalling r_L from equation 10 thus implies

$$\frac{\partial \ell_H}{\partial L} = -\frac{n}{N}\left[\frac{-\frac{(H-L)}{t}[\varpi_L(\ell_L) + \varpi_L\prime(\ell_L)\frac{\partial \ell_L}{\partial L}L] + r_H - r_L}{(H-L)^2}\right]$$

Now

$$\frac{\partial \ell_L}{\partial L} = \frac{n}{N}\left[-\frac{r_H - r_L}{(H-L)^2}\right]$$

And multiplying through by:

$$\frac{t(H-L)}{t(H-L)}$$

Yields:

$$\frac{\partial \ell_H}{\partial L} = -\frac{n}{N}\left[\frac{-\frac{(r_H-r_L)t}{(H-L)} - [\varpi_L(\ell_L) + \frac{\varpi_L\prime(\ell_L)n(r_H-r_L)L}{N(H-L)^2}]}{t(H-L)}\right]$$

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Recall $U = \frac{r_H - r_L}{H - L}$, thus:

$$\frac{\partial \ell_H}{\partial L} = -\frac{n}{N} \left[\frac{\underline{t}U - \varpi_L(\ell_L) - \frac{nL\varpi_L\prime(\ell_L)U}{N(H-L)}}{\underline{t}(H - L)} \right]$$

Implying:

$$\frac{\partial \Pi_H}{\partial L} = -r_H A_H \frac{n}{N} \left[\frac{\underline{t}U - \varpi_L(\ell_L) - \frac{nL\varpi_L\prime(\ell_L)U}{N(H-L)}}{\underline{t}(H - L)} \right] < 0 \qquad (10)$$

Because $r_H, A_H, U, L, n, N, \varpi_L(\ell_L), \underline{t}, (H - L) > 0$, given $(A2)$, that is, $\underline{t}U > \varpi_L(L)$ and recalling $\varpi_L\prime(L) < 0$ the entire term in square brackets is positive. Consequently, $\frac{\partial \Pi_H}{\partial L} < 0$.

Proof of Proposition 7:

$$\frac{\partial \Pi_L}{\partial H} = r_H A_H \frac{\partial \ell_L}{\partial H}$$

Substitute for $r_H = \frac{\varpi_H(\ell_H)}{1 - rr_H}$ to get:

And, thus

$$\frac{\partial \ell_L}{\partial H} = \frac{n}{N} \left[\frac{\frac{(H-L)}{\underline{t}}[\varpi_H(\ell_H) + \varpi_H\prime(\ell_H)\frac{\partial \ell_H}{\partial H}H] - (r_H - r_L)}{(H - L)^2} \right]$$

Now

$$\frac{\partial \ell_H}{\partial H} = \frac{n}{N} \left[\frac{r_H - r_L}{(H - L)^2} \right]$$

And multiplying through by:

$$\frac{\underline{t}(H - L)}{\underline{t}(H - L)}$$

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Yields:

$$\frac{\partial \ell_L}{\partial H} = \frac{n}{N} \left[\frac{\frac{\varpi_{H}{}'(\ell_H)n(r_H - r_L)H}{N(H-L)^2} + \varpi_H(\ell_H) - \frac{(r_H - r_L)\underline{t}}{(H-L)}}{\underline{t}(H-L)} \right]$$

Recall $U = \frac{r_H - r_L}{H - L}$, thus:

$$\frac{\partial \ell_L}{\partial H} = \frac{n}{N} \left[\frac{\frac{nH\varpi_{H}{}'(\ell_H)U}{N(H-L)} + \varpi_H(\ell_H) - \underline{t}U}{\underline{t}(H-L)} \right]$$

Implying:

$$\frac{\partial \Pi_L}{\partial H} = r_L A_L \frac{n}{N} \left[\frac{\frac{nH\varpi_{H}{}'(\ell_H)U}{N(H-L)} + \varpi_H(\ell_H) - \underline{t}U}{\underline{t}(H-L)} \right] < 0 \qquad (11)$$

Because $r_L, A_L, U, L, n, N, \varpi_H(\ell_H), \underline{t}, (H-L) > 0$, given $(A2)$, that is, $\underline{t}U > \varpi_H$ and recalling $\varpi'(H)H < 0$ the entire term in square brackets is negative. Consequently, $\frac{\partial \Pi_L}{\partial H} < 0$.

This is an indication that the high quality bank has an incentive to encourage the low quality institution to remain a low quality firm. However, the reverse is not true, in that, the low quality bank would not encourage the high quality bank to continue to produce a high quality product or even improve upon this. Instead, the low quality bank would want to encourage the high quality type to reduce the quality of its product. As per discussions in Tirole (1988) each firm would increase its market share by coming closer together, however, by distinguishing itself each firm garners a strategic advantage in the market power it gains. Clearly then, one can see that the two types could each benefit from a different source. That is, high quality types benefit from the strategic advantage gained by producing the superior product. On the other hand, if the assumption $\varpi_t(t) < \underline{t}U$ is true the low quality type benefits from increased market share.

5 Conclusion

In a duopoly model of banking product differentiation where both loan and depositor risks as well as increasing returns to scale (IRS) and variable APRs are incorporated several interesting results come to the fore. To review some specifics of the model the IRS term entered as a function of the risk coefficients; a suggestion of Berger and Mester (1997) and Mester (1996), among others. Some of the main results are that under the premise of reserve requirement adjusted loan pricing or the imposition of Usury Laws the high type bank will always advertise a higher minimum value of the interest rate on loans than does the low quality-type bank. Furthermore, the difference in teaser rates/advertised minimum loan prices is an increasing function of the distance between bank types; and, these interest rates themselves are also increasing in the distance between types.

Additionally, comparative statics analysis shows that it is optimal for banks to maximally differentiate themselves, but with a few caveats. It is shown that under the regularity condition, that both banks have positive market share, $(A1)$ and $(A2)$, it is unequivocally true that the high quality-type bank prefers maximal product differentiation. This is so because high quality-types benefit more from the strategic advantage gained through increased market power when it distinguishes itself, versus the cost it incurs to create this distinction.

To the contrary, low quality-type banks may benefit more from increased market share if it does not have to undergo the cost of acquiring higher quality. As such, both the low quality bank and the high quality bank would encourage the other type to reduce its quality. However, this similar action, of course, would result in different outcomes. That is, ceteris paribus, reduced quality on the part of the already low quality bank means accentuated differentiation; but reduced quality on the part of the high quality type attenuates product differentiation.

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